How to Ignore Counterexamples: Homogeneity Across Times and Worlds

by

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This thesis is dedicated to my parents, Ghazala and Asif.

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Abstract

What is the best logical form for sentences involving iteration over possibilities, times or events? Which constructions should be analyzed as involving covert universal quantifiers over abstract objects (like worlds or events) and which constructions would be better analyzed as instances of plural predication, with no covert quantifier?

To gain ground on these questions, I present three case studies in which the theory of homogeneity and non-maximality, originally developed for plural definite noun phrases, is applied to other problem areas: conditionals, temporal adverbs, and weak necessity modals. I give an explicit formal description of what it would mean to treat such objects as plural predication structures, and I provide novel arguments in favor of the plural referential analysis.

For conditionals, the first case study shows that variably strict theories give incorrect results for certain Sobel sequences, and demonstrates how the problem can be fixed by a plural, referential analysis of bare conditionals. In the case of temporal adverbs and habituals, the second case study highlights data that shows that their exception tolerance is sensitive to the Question Under Discussion (QUD). And for weak necessity modals, the third case study gives evidence for their homogeneous behavior under negation, highlighting certain parallels with plural predication.

The conclusion is that, in each case—bare conditionals, bare habituals, temporal adverbs, and weak necessity modals—there are substantial arguments in favor of a non-quantificational analysis. These findings effectively limit the use of silent quantifiers as an analytical tool. In each case where a silent or covert quantifier might seem appropriate, it is worthwhile to consider a

plural referential analysis as an alternative, and to test for the properties of homogeneity and non-maximality.

In addition to this methodological point, these case studies also serve to further our understanding of the diverse constructions under analysis, and reveal new ways in which the denotational domains of times, worlds, events, and individuals are parallel to each other.

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1 Introduction: Homogeneity and Non-maximality

1.1 Introduction

Conditionals, temporal adverbial constructions, and habituals are typically analyzed as involving silent operators that quantify over abstract objects. In conditionals, like (1), the *if*-clause functions as the restrictor for a silent modal, which is a universal quantifier over possible worlds [Kratzer 1981] or situations [Kratzer 1989].[1]

(1) I call a cab if the buses leave early.

In sentences modified by temporal adverbial *when*-clauses, like (2a), or temporal prepositional phrases, like (2b), the adverb is assigned the meaning of a restricted universal quantifier over times or events (e.g. Rothstein 1995).

(2) a. The bees get busy when Spring arrives.

b. Everyone goes to the beach on a day like this.

An alternative to the universal quantifier analysis is a referential analysis. On a referential analysis sentences such as (1-2) contain no universal quantifier over times, events, or worlds.

[1]See Kratzer (2012: p. 64).

Instead, the adverbial modifiers are referring expressions denoting pluralities of times, events, or worlds. For example, the *if*-clause in (1) directly saturates an argument of the predicate. The predicate applies directly to a plurality of worlds, just as a predicate may apply to a plurality of individuals.

In this dissertation, I argue that all of these constructions should be analyzed as referential, rather than quantificational, building on ideas from Križ [2015].[2] The main evidence for this conclusion comes from homogeneity and non-maximality, two characteristic properties of plural definite noun phrases that are also found with all three construction types listed above.

1.2 Defining Homogeneity

Homogeneity is a property of plural predication: A sentence composed of a homogeneous predicate and a plural argument is true if and only if the predicate is true of all parts of the plurality, and false if and only if it is false of all parts of the plurality.[3]. Otherwise, the sentence has an intermediate status, modeled by a third truth value $\star$. For example, in (3), the predicate outside takes the term $\bigoplus$ kid as an argument (the mereological sum of all elements in the extension of kid).

(3) The kids are outside. $\mathsf{outside}(\bigoplus \mathsf{kid})$

Homogeneous plural predications also display **non-maximal** readings. A plural predication like (3) is non-maximal if it has intermediate status, but can still be accepted as true depending on the discourse context in which it is interpreted. Specifically, a discourse participant may accept the intermediate plural predication as true enough if they do not consider the exceptions

[2]In particular, Križ argues for a referential treatment of conditionals and generics in Križ (2015: Ch. 7). Some earlier work advocates a referential analysis of conditionals include Bittner [2001] and Schlenker [2004]. For generics, [Sterken 2015] argues that the logical forms of generics involve silent demonstratives, though she does not discuss plurality or homogeneity. I know of no previous referential analyses of temporal adverbs.

[3]Earlier work on homogeneity treats it as a semantic presupposition of plural definite NPs, such as Löbner [2000] and Gajewski [2005]

relevant to their informational goals, or if they are operating under some non-maximal standard for satisfying the predicate.

For example, if a baby is known to sleeping inside, then the baby's location may not be relevant to the informational goals of the speaker, in contrast to the other kids. The speaker may utter (3) despite knowing the baby is inside, and others possessing the same information may treat this utterance as true, if they share the speaker's informational concerns. Or, suppose dinner is ready, and the question under discussion is whether *anyone* is still outside, and needs to be called in. Then, even if some kids are inside and some are outside, the speaker might use (3) regardless.

Non-maximality for plural NPs was first discussed in these terms by Malamud [2012], who broke new ground by giving a formal pragmatic analysis of the use conditions of plural definites in terms of decision theory.[4] Križ's (2015) dissertation was the first work to connect Malamud's formal pragmatic model of non-maximality to homogeneity, and the first to suggest that non-maximality plays a role in the exception-tolerance of bare conditionals and generics, rather than being a property of predicates of individuals alone.

1.3 Rethinking Quantity and Relevance

In Križ's unified framework, non-maximality is formally implemented in two steps. At the level of semantics, plural predication structures are assigned an indeterminate truth value (written as $\star$) in any context where homogeneity is not satisfied. The intermediate truth value is then repaired to *true enough* or *false enough* via a pragmatic algorithm called Sufficient Truth. For Križ, assertability requires Sufficient Truth [Križ 2016] rather than strict truth. Sufficient Truth is truth relativized to a partition over possible worlds, called the Current Issue. In this work, I refer to this partition as the Question Under Discussion (QUD), a related contextual parameter

[4]Important work on non-maximality prior to Malamud uses very different language in the framing. This includes Krifka [1996], which derives plural definite meanings using the Strongest Meaning Hypothesis, and Szabolcsi & Haddican [2004], which takes a more general view linking plurals to conjunction.

first introduced by Roberts [2012].[5] A proposition p is *true enough* at w if it is true at some world u that lies in the same equivalence class as w, even if p is indeterminate ($\star$) at w itself. Note that if p is false at w, it cannot be true enough at w. Only indeterminate (or true) propositions can be true enough.

Previous work in formal semantics and pragmatics has, either explicitly or implicitly, taken a speaker's belief in the truth of a proposition p to be a precondition for asserting p in information-seeking discourse. Križ's theory of non-maximality establishes a weaker condition, namely Sufficient Truth. This marks an interesting departure from the standard view of truth and assertion. For example, the Gricean tradition in pragmatics [Grice 1975] conceives of the relationship between truth and assertability as follows. From the start, we are required to assert only what we believe to be true (Maxim of Quality). However, truth is necessary but not sufficient to guarantee assertability. Utterances must also make assertions that are relevant (Maxim of Relevance). Their contents should be more informative than the contents of other possible true assertions (Maxim of Quantity). And utterances should be designed to be concise, and to minimize the possibility for misunderstanding (Maxim of Manner).

Grice's maxims are individually plausible, but in practice they often pull in different directions. Some progress has been made in adjudicating between Maxims in cases where they conflict. For example Roberts's (2012) Question Under Discussion theory of discourse makes predictions about how speakers balance Quantity and Relevance. And indeed, since Roberts, implicit Questions Under Discussion have sometimes been taken to provide the very alternatives against which Quantity is evaluated [Spector 2007].

From this perspective, Križ's framework can be seen as a theory of how Relevance interacts with Quality. Rather than treating Relevance as merely a filter on true utterances, Sufficient

[5] For a discussion of the difference between Current Issues and QUDs, see Križ (2015: p. 85). Križ chooses not to identify the two notions because certain examples show that the last-asked question does not license the readings that would be expected if that question were the same as the Current Issue. These examples are not directly relevant for the present study, so I choose to simplify the discussion by using QUDs throughout.

Truth allows relevant assertions to meet Quality even if they are not strictly true. Thus, Quality and Relevance are intertwined in a way not previously thought possible, and this constitutes a significant shift in perspective for theorists of the semantics-pragmatics interface.

If Križ's hypothesis is right, then the implications for empirical methodologies in linguistics are potentially significant. In particular, felicity judgments elicited without carefully controlling the QUD might turn out to be less informative than previously thought. For this reason, understanding the constraints on exception tolerance across all domains in semantics should be considered an important growing area of research. This dissertation contributes to that project by testing the limits of Križ's theory. Here, I focus on expressions that do not (transparently) refer to plural objects, and give reasons to prefer a plural referential analysis over other alternatives.

In the remainder of Chapter 1, I will review the predictions of the Sufficient Truth theory for plural definites, to prepare the readers for further applications.

1.4 Homogeneity and overlap

The theory of [Križ 2016] explains homogeneity via the following formal principle. I call this principle Homogeneity in a different font to distinguish the theoretical notion from the theory-neutral term homogeneity.

(4) Homogeneity

Define the positive extension of a predicate P as $[\![P]\!]^+ = \{x \mid P(x) = 1\}$, and the negative extension $[\![P]\!]^- = \{x \mid P(x) = 0\}$. A predicate P is **homogeneous** if no element in $[\![P]\!]^-$ overlaps with any element in $[\![P]\!]^+$.

Informally, (4) defines homogeneity as a property of predicates. A predicate is homogeneous if it never assigns opposite truth values to two possible arguments that overlap.[6] However, a homogeneous predicate is free to assign $\star$ to any element in its domain. Homogeneous distributive

[6] A complete account would require Homogeneity to be stated for n-place predicates.

predicates have the additional requirement that $P(x) = 1$ only if every atom in y in x also has $P(y) = 1$. Homogeneous collective predicates may have only just a single non-atomic plurality in their positive extension, depending on the model.

In the present framework, conjunction, disjunction, negation, and existential and universal quantifiers all obey a Strong Kleene semantics. Thus, quantified sentences with plural definites in the scope of a quantifier have the following truth and falsity conditions. Throughout these examples, the subscript D is a domain restriction, modeled as a free variable that takes its value from the assignment function $\mathfrak{a}$. (I assume that quantifiers also have domain restrictions, though they are not important for present purposes.)

$$[\![\text{abby} \cdot (\text{read} \cdot (\text{the}_D \cdot \text{books}))]\!]^{\mathfrak{a}}(w) = \begin{cases} 1 & \text{Abby read all}_D \text{ books in } w \\ 0 & \text{Abby read no}_D \text{ books in } w \\ \star & \text{otherwise} \end{cases}$$

$$[\![\text{everyone} \cdot (\text{read} \cdot (\text{the}_D \cdot \text{books}))]\!]^{\mathfrak{a}}(w) = \begin{cases} 1 & \text{everyone read all}_D \text{ books in } w \\ 0 & \text{someone read no}_D \text{ books in } w \\ \star & \text{otherwise} \end{cases}$$

$$[\![\text{no one} \cdot (\text{read} \cdot (\text{the}_D \cdot \text{books}))]\!]^{\mathfrak{a}}(w) = \begin{cases} 1 & \text{no one read any}_D \text{ books in } w \\ 0 & \text{someone read all}_D \text{ books in } w \\ \star & \text{otherwise} \end{cases}$$

Homogeneity can also be removed by the presence of explicit quantifiers like *all*. We can test homogeneity removal by seeing whether certain followups produce a contradiction. For example, the followup in (5a) sounds odd, which indicates that the first clause is homogeneous.

(5) a. ?? Abby didn't read the books, she read two of them.

b. Abby didn't read all the books, she read two of them.

c. Abby didn't read most of the books, she read two of them.

But the followups in (5b) and (5c) do not seem contradictory. The quantifier *all* behaves like a normal universal quantifier under negation, but (5a) is scopeless. As we will see in each case study, there are also quantifiers that act as homogeneity removers in the domains of times and worlds.

1.5 Non-maximality and Sufficient Truth

Having seen the theory behind Homogeneity in plural predication, we can now introduce the Sufficient Truth theory (ST). Non-maximal readings arise when sentences that are indeterminate manage to be sufficiently true due to the QUD.

(6) Sufficient Truth (ST) Križ [2016]

We write $\simeq_I$ for the equivalence relation that holds of two worlds u, v iff u and v are in the same cell of an issue I. A sentence S is **true enough** in world w with respect to I iff there is some world w' such that $w' \in [\![S]\!]^+$ (S is literally true in w') and $w \simeq_I w'$.

On its own, Sufficient Truth makes no new predictions about felicity. What we need is pragmatic principles that are sensitive to Sufficient Truth rather than strict truth. For this reason, Križ defines the Weak Maxim of Quality as follows.

(7) Weak Maxim of Quality Križ [2016]

Say only sentences which you believe to be true enough.

We need one further principle, which guarantees that whenever a proposition S is used to address and issue I, I must not be indifferent to the truth of S. This is simply a special case of Grice's Maxim of Relevance, formalized to suit Križ's needs.

(8) Addressing an Issue Križ [2016]

A sentence S may not be used to address an issue I if any cell $i \in I$ overlaps with both the positive and the negative extension of S, i.e. if S is true in some worlds in i and false in others.

This principle is crucial for explaining why it is infelicitous to deny a true-enough proposition. It also explains why a false sentence can never be true enough.

With all this machinery in place, we examine the predictions for plural definites. For example, suppose a set of students are given a reading list, and told that they must read at least half of the books to pass their class. If the salient issue is *Who passed?*, then ST predicts that the following utterances (9-11) are all true enough in their given contexts, assuming the meanings of each sentence are those generated by the trivalent homogeneous predicates described above.

(9) *Context: Students must read at least half the books on the reading list to pass. Abby read most, but not all, of the reading list books.*

Abby read the books.

(10) *Context: As before, but this time all the students read at least half.*

Every student read the books.

(11) *Context: As before, but this time all the students read fewer than half.*

No student read the books.

For further evidence, we can compare responses with *well* to denials with *no*. Under ST, if a sentence S is indeterminate at w, but true enough with respect to issue I, then $\neg S$ will fail to be true enough, and is therefore predicted to be infelicitious due to the Weak Maxim of Quality.

To see why negation works this way in ST, we need to combine the predictions of ST with the Relevance filter imposed by Addressing an Issue in (8). Suppose u is an S-world ($u \in [\![S]\!]^+$)

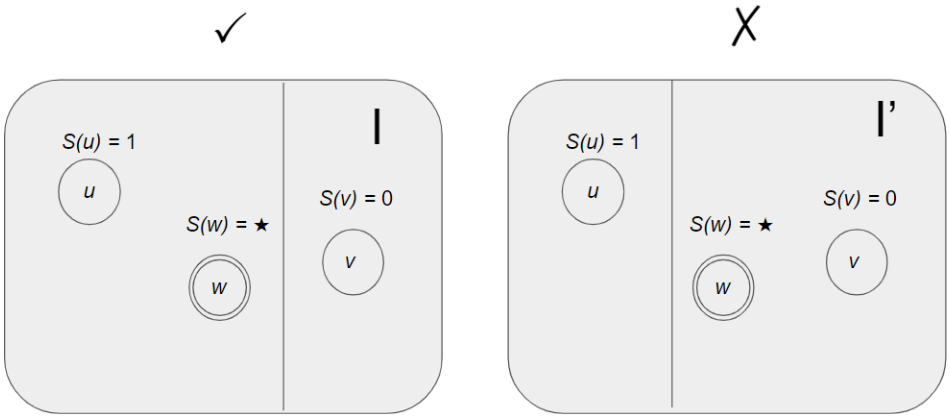

Figure 1.1: S indeterminate at w. In the Issue I on the left, S true enough at w, since $w \approx_I u$. On the right, S is not true enough relative to issue I'.

and the evaluation world w is I-equivalent u ($w \approx_I u$). This is precisely what it means for S to be true enough relative to I. This is depicted graphically in Figure 1.1.

We require that S Addresses the Issue I—see (8)—which entails that no $\neg S$-world may be I-equivalent to the S-world u. Thus, whenever S is true enough relative to I, its negation $\neg S$ cannot be true enough relative to I . Thus, the Weak Maxim of Quality, combined with ST and Addressing an Issue, predicts the following contrasts.

(12) *Context: Students must read at least half the books on the reading list to pass. Abby read most, but not all, of the reading list books, say 7/10.*

A: Abby read the books.

B: Well, Abby read most of them.

B': # No, Abby read most of them.

(13) *Context: As before, but this time all the students read at least half.*

A: Every student read the books.

B: Well, they read most of them.

B': # No, they read most of them.

(14) *Context: As before, but this time all the students read between 1/10 and 4/10.*

A: No students read the books.

B: Well, they read some of them.

B': # No, they read some of them.

To summarize, the exact predictions of Sufficient Truth depend both on the precise nature of the QUD and on the contextual assumptions in place. If the QUD were instead *Who read all the books?*, then (9-11) would be infelicitous, and their negations would be felicitous. This is because, relative to the partition induced by the question *Who read all the books?*, a world where (say) Abby read all the books is not equivalent to any world where Abby read only some of the books.

In each section of the dissertation, I apply the framework described in the previous section to a variety of referential expressions. Some, like *if*-clauses and temporal adverbs, are overt. Others are silent, such as the temporal and modal domain restrictions in habitual and generic sentences.

1.6 Dissertation roadmap

In this chapter I have laid out the goals of the dissertation, and introduced the formal framework from Križ [2015]. The structure of the dissertation is as follows. After the introduction, I present three separate applications of the Sufficient Truth theory, to conditionals, temporal adverbs, and weak necessity modals. In each case, I explore how Sufficient Truth predicts patterns of exception tolerance and intolerance that depend on the QUD.

In Chapter 2, I show that the Sufficient Truth theory of conditionals has an important advantage over the variably strict theory of conditionals. The referential approach with homogeneity resolves a serious (but little-known) problem with the standard account of bare conditionals, namely that Sobel sequences with bare conditionals are predicted to have incorrect entailments [Willer 2017]. The variably strict theory of counterfactuals (as found in Stalnaker 1968, Lewis 1973, Kratzer 1986, and subsequent work) predicts that Sobel sequences entail the falsity of their conditional antecedents. This prediction is false when we extend the theory to bare indicative conditionals, and it is even false for certain counterfactuals, those that lack the entailment that their antecedents are false [Anderson 1951]. The Sufficient Truth theory avoids these problems.

In Chapter 3, I argue that imprecise readings of English bare habitual sentences and temporal adverbs should be seen as non-maximal readings. I develop an idea from [Ferreira 2005] that habituals involve plural reference to times, and show that this idea implies a pragmatic theory of imprecise habituals based on Sufficient Truth. I show that this theory has advantages over alternative semantic accounts, such as [Deo 2009].

In Chapter 4, I present the results of joint work with Paloma Jeretič, in which we develop a novel analysis of weak necessity modals like English *should*. In the analysis, *should* is a plural referential expression that picks out a plurality of modally accessible worlds. We show that this analysis uniquely captures the behavior of *should* under both same-clause and higher-clause negation. We account for the apparent weakness of weak necessity modals as a type of non-maximality, and we compare the non-maximality analysis to previous work on the topic.[7]

In Chapter 5, I conclude my investigation by summarizing the results, and describing some useful directions for further research.

[7] This work was originally published in Agha & Jeretič [2022]. I have obtained the rights to reproduce this work here. The version here is a bit different in detail from the original in its presentation, but the analysis remains unchanged.

2 Conditionals, Non-maximality, and Sobel Sequences

2.1 Introduction

Since Kratzer [1981], much work in formal semantics has assumed that all conditionals have a tripartite quantificational structure, in which a modal operator takes the *if*-clause as its restrictor, and takes the main clause as its scope argument. However, the uniform quantificational treatment of all conditionals faces significant problems. I argue, following Križ, that these problems are best addressed in English by adopting a plural referential treatment of bare conditionals, as well as conditionals under *will/would*, while maintaining the standard quantificational treatment of other explicitly modalized conditionals.

In English, three broad classes of conditionals must be distinguished. First, we have explicitly modal conditionals like those in (15). Second, we have bare indicative conditionals like (16). Under the mainstream quantificational theory of conditionals, bare conditionals are headed by a silent epistemic necessity modal (Kratzer 1991: p. 654).

(15) Modal conditionals

a. The junior senator **has to** vote yes if the senior ones do.

b. The junior senator **necessarily** votes yes if the senior ones do.

(16) Bare conditionals

a. The junior senator votes yes if the senior ones do. (bare, non-past)

b. The junior senator voted yes if the senior ones did. (bare, past)

Third, we have conditionals like those in (17), which are headed by *will* and its subjunctive counterpart *would*. Some readers might be puzzled by the distinction between explicitly modalized conditionals and conditionals headed by *will/would*, since the latter auxiliaries are sometimes treated as modal operators (e.g. Kaufmann 2005). But as we will see, conditionals like those in (17) turn out to pattern with bare conditionals when tested for homogeneity effects.

(17) *Will/would* conditionals

a. The junior senator **will** vote yes if the senior ones do. (*will*-conditional)

b. The junior senator **would** vote yes if the senior ones did. (*would*, non-past)

c. The junior senator **would have** voted yes if the senior ones had. (*would*, past)

The uniform quantificational treatment of all of these types of conditionals faces two related problems, each of which I discuss in detail in the sections below. First, bare conditionals are scopeless with respect to both same-clause and higher-clause negation, in contrast to other modals.[1]

(18) a. The junior senator did not vote yes if the senior ones did...

...but she might have.

b. The junior senator did not **necessarily** vote yes if the senior ones did...

✓...but she might have.

In other words, bare conditionals appear to be homogeneous in their interactions with negation, a connection first made by von Fintel [1997]. This can be accounted for under a referential

[1]The same applies to conditionals headed by *will/would*, though I delay discussion of these cases until later.

analysis of *if*-clauses, as suggested by Križ (2015: Ch. 7). I review and extend Križ's argument below.

Second, bare conditionals are acceptable in Sobel sequences, as seen in (19).

(19) a. If I get a cat, I'll be happy.

b. But if I get a cat and my landlord kicks me out, I won't be happy.

To maintain the uniform quantificational theory, bare conditional Sobel sequences have to be explained using the same strategy as counterfactual Sobel sequences. However, extending the theory of counterfactual Sobel sequences in this way leads to incorrect predictions. It turns out that this is not a problem for the referential analysis. The account of Sobel sequences within the referential analysis of conditionals is the main novel contribution of this chapter.

2.2 Sobel sequences and the puzzle for strict conditionals

The sequences of conditionals in (20a-b) and (21a-b) are examples of Sobel sequences.[2] Sobel sequences follow a common schema, as in (22).

(20) a. If I get a cat, I'll be happy.

b. But if I get a cat and my landlord kicks me out, I won't be happy.

(21) a. If Annie went to the party, then Connie went too.

b. But if Annie and Bonnie went, then Connie didn't go.

(22) (i) ⌜if A, C⌝

(ii) ⌜if A and B, not C⌝

[2] Lewis [1973] attributes examples like this to Howard Sobel, hence the name.

The characteristic property of Sobel sequences is that first sentence (ii) ⌜if A, C⌝ appears to require that all A-worlds must be C-worlds, while the second sentence (ii) ⌜if A and B, not C⌝ appears to contradict that generalization, saying that some A-worlds (namely $A \wedge B$-worlds) are not C-worlds.

For example, (20a) says that all worlds where the speaker gets a cat are worlds where they will be happy,while (20b) appears to contradict this claim. Example (20) is composed of future indicative conditionals, while (21) is composed of bare past tense conditionals. But the classic examples of Sobel sequences involve past tense *would* conditionals like in (23), typically with a counterfactual interpretation.

(23) *Context: Neither Annie, Bonnie, or Connie actually went to the party.*

a. If Annie had gone to the party, then Connie would have gone too.

b. But if Annie and Bonnie had gone, Connie would have stayed home.

The felicity of Sobel sequences is only compatible with some semantic theories of natural language conditionals. In particular, it is incompatible with the so-called *strict theory* of conditionals. According to the strict theory, a conditional of the form ⌜if A, C⌝ has the same truth conditions as the modal logic formula $\Box(A \rightarrow C)$, composed of a material conditional under a unary necessity modal.

On the strict theory, the conditional (i) is true only if every accessible A-world is a C-world. The strict theory cannot accommodate Sobel sequences, assuming conditionals (i) and (ii) rely on the same modal accessibility relation. (There are also dynamic strict theories that allow sequences of conditionals to be evaluated on different sets of accessible worlds, and those theories permit Sobel sequences to be felicitous. I discuss dynamic strict theories in section 2.9.)

The standard approach to counterfactual Sobel sequences is to abandon the strict theory in favor of a more complex conditional semantics. However, as we will see, the standard solution cannot be extended to bare indicative conditionals without predicting that Sobel sequences have a false entailment. Thus, after reviewing the standard solution, we will return to the strict theory, and see how the theory of homogeneity and non-maximality can offer less problematic alternative.

2.3 The variably strict approach to Sobel sequences

The *variably strict* approach for counterfactual Sobel sequences is to assume that the modal quantifiers in conditionals (i) ⌜if A, C⌝ and (ii) ⌜if A and B, not C⌝ each quantify over distinct sets. By the variably strict approach, I mean the shared strategy of all variably strict theories of conditionals, including Stalnaker [1968], Lewis [1973], and Kratzer [1986].

In variably strict theories, it is possible for the sets quantified over in conditionals (i) and (ii) to be disjoint, despite the fact that the set of $A \wedge B$-worlds is a subset of the A-worlds. Unlike static strict theories, variably strict theories make it possible for the two quantifier domains in (i) and (ii) to be non-overlapping. This is possible because conditional (i) does not quantify over all accessible A-worlds, only a special subset of them. Similarly, conditional (ii) only quantifies over a special subset of the accessible $A \wedge B$-worlds.

For any given conditional, I will call this special subset the *modal domain* of the conditional. Different theories compute the modal domain of a conditional in different ways, but all variably strict theories allow the possibility that Sobel sequences can have disjoint modal domains, and they all predict that Sobel sequence conditionals are consistent only if their modal domains are disjoint. In summary, variably strict conditionals quantify over a modal domain, a proper subset of the *if*-clause proposition, allowing Sobel sequences to be consistent as long as the two modal domains are chosen to be non-overlapping.

2.3.1 Formal details

Each variably strict theory takes different model-theoretic objects as primitive. For Stalnaker [1968], there is a selection function f that picks out the unique closest A-world (if it exists), so Stalnaker's modal domain is either a singleton set or the empty set.[3] For Lewis [1973], the domain of closest A-worlds (if it exists) is the set of A-worlds that lie within the smallest sphere that contains A-worlds.[4] Lewis's notion of "closeness" is determined by a similarity ordering relative to w. For Kratzer [1986], the modal domain is the subset of the modal base that satisfies the most constraints in the ordering source $g(w)$.

To abstract away from some of these details, I adopt Stalnaker's way of talking: Assume the existence of a selection function f, whose job is to map a world w and a proposition A directly to its modal domain $f(w, A)$. The modal domain is interpreted as the set of A-worlds that are closest to w. Unlike Stalnaker, I use a "plural" version of selection functions, where $f(w, A)$ may be a non-singleton set, following Schlenker [2004]. The selection function f is a contextual parameter, akin to Kratzer's modal base and ordering source, that effectively delivers the output of both. Figure 2.1 diagrammatically represents $f(w, A)$, the modal domain of (i) ⌜if A, C⌝ is true at w. In this case, (i) is true at w, and w is not an A-world.

Selection functions satisfy certain constraints that restrict the possible meanings of conditionals. These constraints are somewhat technical and not entirely relevant. I refer the reader to Schlenker (2004: pp. 430-438) for a detailed discussion. The most important constraint for present purposes is Weak Centering, which I introduce in Section 2.3.4.

[3]Strictly speaking, for Stalnaker (1968: p. 46) the selection function picks out a either single world or the "absurd world."

[4]This formulation presupposes the Limit Assumption, corresponding to "Analysis 2" on (Lewis 1973: p. 61). I set aside the later version of Lewis's theory that rejects the Limit Assumption (p. 63).

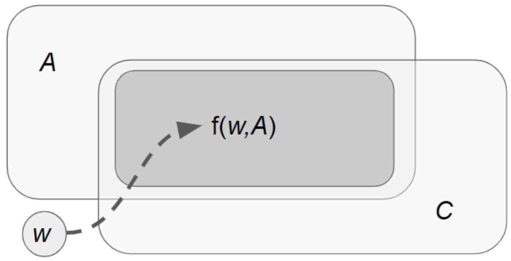

Figure 2.1: The Modal Domain
When we evaluate ⌜if A, then C⌝ at the world w, the set $f(w, A)$ is its modal domain. In this model, the modal domain lies within C, and so the conditional is true.

2.3.2 Sobel sequences require disjoint modal domains

Recall that under the strict theory, all Sobel sequences are inconsistent, since the set of accessible A-worlds contains the set of accessible $A \wedge B$-worlds.[5] Under the variably strict theory, the modal domains for (i) and (ii) are $f(w, A)$ and $f(w, A \cap B)$ respectively, and these need not overlap. In (24), it is shown that for the Sobel sequence to be consistent, they *must* not overlap.

(24) Fact: Consistent Sobel sequence conditionals must have disjoint domains

Let w be any world, and A, B, C be three arbitrary propositions. Suppose w is such that the pair of conditionals (i) and (ii) are both true at w.

(i) ⌜if A, then C⌝

(ii) ⌜if A and B, then not C⌝

Then it must be the case that $f(w, A) \cap f(A \cap B) = \emptyset$.

Proof. By contradiction: Let u be a world in $f(w, A) \cap f(A \wedge B)$. Suppose (i) and (ii) are true. By (i), u must be a C-world. But by (ii), u must be a $\neg C$-world. By contradiction, $f(w, A) \cap f(w, A \cap B)$ must be empty.

This idea may be easier to appreciate in a diagram. Figure 2.2 shows a situation in which the Sobel sequence conditionals are both true. In the figure, w is outside A, B, and C. The modal

[5]More precisely, if $R(w)$ is the set of accessible worlds, it is impossible to satisfy both $(R(w) \cap A) \subseteq C$ and $(R(w) \cap A \cap B) \subseteq W \setminus C$ unless $R(w) \cap A \cap B = \emptyset$.

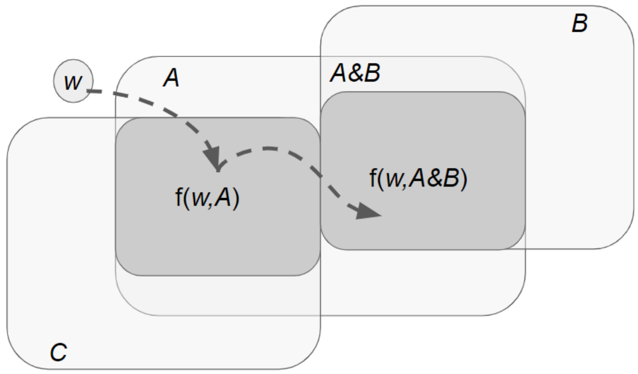

Figure 2.2: Disjoint Domains

When the closest A-worlds from w are not B-worlds, the domains of (i) and (ii) can be disjoint.

(i) ⌜if A, then C⌝ $\qquad f(w, A) \subseteq C$

(ii) ⌜if A and B, then not C⌝ $\qquad f(w, A \cap B) \cap C = \emptyset$

domain of (i) is contained in C, and does not overlap B, and the modal domain of (ii) does not overlap C. This is a case where the closest $A \wedge B$-worlds from w are totally distinct from the closest A-worlds from w.

This corresponds to our intuitions about specific cases, such as (25).

(25) *Context: Annie is friends with both Bonnie and Connie, and Connie is friends with Annie, but Connie is avoiding Bonnie.*

a. If Annie went to the party, Connie went too.

b. But if Annie and Bonnie went to the party, Connie didn't go.

If Connie is avoiding Bonnie, then Connie goes in none of the worlds where Bonnie goes, regardless of what Annie does. So the closest worlds where Annie and Bonnie go look very different from the closest worlds where Annie goes alone.

2.3.3 Epistemically open conditionals in Sobel sequences

Conditionals with epistemically possible antecedents are problematic for variably strict approaches to Sobel sequences, a fact which was not appreciated before Willer [2017]. The literature on Sobel sequences has mostly focused on subjunctive conditionals, and usually assumes that these conditionals are interpreted counterfactually.[6] Such discussions do not explicitly deal with conditionals in contexts where the *if*-clause propositions are epistemically possible, rather than assumed false. Conditionals with possible antecedents, which I will refer to as *epistemically open conditionals*, pose a theoretical problem because variably strict approaches to Sobel sequences cannot account for their epistemic openness.

I first describe the empirical picture, before explaining why variably strict approaches fall short. Consider example (26). (26b) raises the possiblity that I get a cat (A) and my landlord kicks me out (B). Suppose that in the context prior to (26a), discourse participants do not know whether my landlord will kick me out. In this particular example, that assumption is plausible.

(26) *Context: Nobody knows whether my landlord will kick me out.*

a. If I get a cat, I'll be happy.

b. But if I get a cat and my landlord kicks me out, I won't be happy.

The relevant empirical question is: What is the status of the proposition $A \wedge B$ in the posterior context? Every native speaker I have consulted about this case (and similar cases) reports that in the posterior context, it is still not determined whether my landlord will kick me out. If anything, the fact that (26b) raises the possibility to salience *increases* its perceived likelihood relative to the prior context.

By contrast, all variably strict theories incorrectly predict that any sequence of the form (i) $\ulcorner$if A, $C\urcorner$, (ii) $\ulcorner$if A and B, not $C\urcorner$ entails that $A \wedge B$ is false. As far as I know, no work prior to Willer [2017] explicitly discusses this incorrect prediction. Once we locate our conditional semantics within a Stalnakerian context model, variably strict theories produce a posterior context set that incorrectly represents the intuitions of native speakers: If a shared context set is updated by

[6]Stalnaker's and Lewis's frameworks were explictly designed for counterfactual conditionals, and Kratzer's most influential papers do not focus on Sobel sequences.

intersection with assertive content, the posterior context after evaluating (26) rules out that I get a cat and my landlord kicks me out. In the next subsection, we will see why.

2.3.4 Epistemically open conditionals cannot have disjoint domains

In Section 2.3.1, we saw some applications of the variably strict theory. In those cases, it was assumed that the evaluation world w is one where $A \wedge B$ is false. Though the variably strict approach works well under those assumptions, the predicted entailments outside of this special case are incorrect. In fact, under the variably strict approach, all Sobel sequences of the form (i) ⌜if A, C⌝; (ii) ⌜if A and B, not C⌝ have a false entailment, namely that $A \wedge B$ must be false. Below, I explain why this is the case, generalizing an argument originally due to Willer (2017: pp. 6-11).

The problem is linked to an essential constraint on selection functions, called Weak Centering. Weak Centering says that the set $f(w, A)$—the modal domain of e.g. ⌜if A, C⌝—contains w whenever A is true in w.

(27) Weak Centering

If $w \in A$, then $w \in f(w, A)$.

(If A is true in w, then w is among the closest A-worlds to w.)

This constraint is essential because it guarantees the validity of Modus Ponens: If both ⌜A⌝ and ⌜if A, C⌝ are true, then so is ⌜C⌝. It is also required for the common interpretation that $f(w, A)$ is the set of "closest" A-worlds to w, since no world is closer to w than itself. Figure 2.3 shows an example where Weak Centering is satisfied (left), and one in which Weak Centering is violated (right).

Now, suppose w is a world where ⌜A and B⌝ is true. The conditional (i) ⌜if A, C⌝ requires that all closest A-worlds are C-worlds. But the conditional (ii) ⌜if A and B, not C⌝ requires that all closest $A \wedge B$-worlds are $\neg C$-worlds. But if w is an $A \wedge B$-world, it must lie in both modal domains, a contradiction.

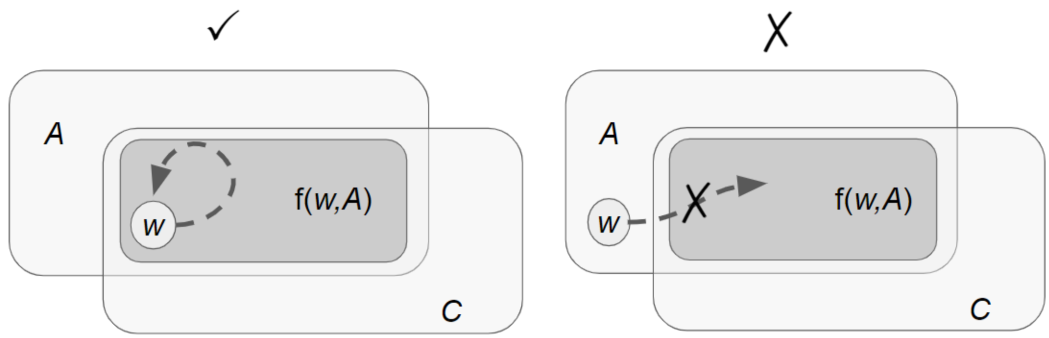

Figure 2.3: Weak Centering
Weak Centering requires that the domain of ⌜if A, then C⌝ must contain the evaluation world w whenever ⌜A⌝ is true in w. This validates Modus Ponens.

So if the prior context is one where $A \wedge B$ is possible, all $A \wedge B$-worlds will be eliminated once the Sobel sequence becomes common ground. As the previous section shows, this the wrong result.

To summarize, under the variably strict approach, a Sobel sequence of the form (i); (ii) is consistent as long as the two conditionals have disjoint modal domains. Due to Weak Centering, Sobel sequence conditionals can only be (both) true at worlds where the antecedent of (ii) is false, because otherwise the modal domains both contain the evaluation world. Thus, Sobel sequences in which the conditionals are interpreted counterfactually pose no problems.

However, indicative conditionals are always used in contexts where their antecedents are assumed possible.[7] This problem is not limited to indicative conditionals. Subjunctive conditionals (headed by *would*) can also be used in non-counterfactual contexts, though such cases are discussed less often than counterfactual ones [von Fintel 1998; Zakkou 2020]. Sobel sequences entail the falsity of the antecedent of (ii), and thus in non-counterfactual contexts they are wrongly predicted to eliminate the antecedent of (ii) from the common ground.

In the next section, we will see how to avoid this problem by assigning a plural referential denotation to *if*-clauses, combined with Križ's theory of homogeneity and non-maximality.

[7] Some theories go as far as to argue that antecedent possibility is a presupposition of indicative conditionals, but I remain neutral on this point.

2.4 The analysis

In principle, there are at least two ways to combine conditionals with Sufficient Truth (ST). We can start from the strict theory, where ⌜if A, C⌝ has the truth conditions of $\Box(A \to C)$, or from the variably strict theory, where a selection function chooses the modal domain of the conditional. I will first outline ST using the strict theory, and later on I will compare this to a more complicated version based on the variably strict theory.[8]

2.4.1 Sufficient Truth for strict conditionals

Consider the strict conditional ⌜if_R A, then C⌝, where R is a domain restriction. Here R is implemented as a function that takes a world w and returns the set of all worlds accessible from w—it can also be thought of as a curried accessibility relation. The *classical* strict conditional is true if and only if all accessible A-worlds are C-worlds, and false otherwise. The trivalent strict conditional, which I adopt here, has the same truth conditions, but has a truth value gap due to homogeneity. The truth and falsity conditions are given in (28), and will be derived compositionally later on.

(28) Trivalent Strict version of (i)

$$[\![\text{if}_R\ A, \text{then}\ C]\!]^{a} = \begin{cases} 1 & \text{if } A \cap R(w) \subseteq C \\ 0 & \text{if } A \cap R(w) \cap C = \emptyset \\ \star & \text{otherwise} \end{cases}$$

The truth and falsity conditions of the second Sobel sequence conditionals are given in (29).

[8] Notice that adopting a strict theory of bare conditionals is not in conflict with Kratzer's framework. Strict conditionals can be simulated in Kratzer's framework by assuming an empty ordering source (Kratzer 1981: p. 66). My goal is to retain the standard Kratzerian semantics for most modals and modal conditionals, while changing the semantics of bare conditionals (and later on, weak necessity modals like *should*).

(29) Trivalent Strict version of (ii)

$$[\![\text{if}_R\ A \text{ and } B, \text{ then } C]\!]^a = \begin{cases} 1 & \text{if } A \cap B \cap R(w) \subseteq C \\ 0 & \text{if } A \cap B \cap R(w) \cap C = \emptyset \\ \star & \text{otherwise} \end{cases}$$

I assume that the restriction R is such that $w \in R(w)$, so that the evaluation world is always accessible from itself. This makes Modus Ponens valid, allowing a fair comparison with the variably strict theory with Weak Centering, which I critiqued in Section 2.3.4. I also assume that changes in the domain restriction, while possible, do not explain Sobel sequences.[9]

This is because, on the current theory, the Sobel sequence conditionals (i) and (ii) can never both be true. Their acceptability is due to Sufficient Truth. The definition of Sufficient Truth is repeated in (30). Sufficient Truth allows a conditional with indeterminate (⋆) status to be accepted as true enough if the evaluation world is not relevantly different from the worlds where the conditional is true. On the strict theory with Sufficient Truth, an acceptable Sobel sequence is one in which both conditionals are true enough.

(30) Sufficient Truth (ST) Križ [2016]

We write $\simeq_I$ for the equivalence relation that holds of two worlds u, v iff u and v are in the same cell of an issue I. A sentence S is **true enough** in world w with respect to I iff there is some world w' such that $w' \in [\![S]\!]^+$ (S is literally true in w') and $w \simeq_I w'$.

Each of the conditionals (i) ⌜if A, C⌝ and (ii) ⌜if A and B, not C⌝ has three possible truth values, and intermediate conditionals may be true enough depending on the QUD. So we need a clear condition that tells us exactly when (i) and (ii) are true enough, depending on the status of A and B (i.e. the facts) and the QUD (i.e. what is relevant).

[9] The idea that the acceptability of a Sobel sequence requires a change in the modal base of the conditionals is equivalent to the Dynamic Strict view, which I discuss later on.

2.4.2 The Limiting Issue

The conditional sequence $\ulcorner$if A, $C\urcorner$; $\ulcorner$if A and B, not $C\urcorner$ is only felicitous when the QUD is too coarse-grained to distinguish between the actual world and the worlds that make (i) and (ii) true. It turns out that we can define something called a Limiting Issue: A Limiting Issue for the sequence (i); (ii) is the coarsest partition L such that:

- Whenever L is not part of the QUD, the sequence (i); (ii) will be (at least) true enough.
- Whenever L is part of the QUD, (i); (ii) will fail to be true enough (so the Sobel sequence will be infelicitous).

(31) **Limiting Issues for Strict Sobel Sequences**

Define the set of optimistic worlds $O = \{w \in K \mid \exists w' \in D(w) \cap A : w' \in B\}$. As before, the Limiting Issue is $L = \{\langle w, w' \rangle \mid w \in O \leftrightarrow w' \in O\}$.

Intuitively, the Limiting Issue is the question of whether any accessible A-worlds are B-worlds.

(32) Limiting Issues For the Sobel sequence

(i) $\ulcorner$if A, $C\urcorner$

(ii)$\ulcorner$if A and B, not $C\urcorner$

Define the *Limiting Issue* for (i); (ii) to be $L = \{\langle w, w' \rangle \mid w \in O \leftrightarrow w' \in O\}$, where the set O is given by

$$O = \{w \in K \mid \exists w' \in D(w) \cap A : w' \in B\}$$

The key claim here is that if the QUD entails the Limiting Issue L, then conditional (i) cannot be true enough. A question P entails Q if whenever $w \approx_P w'$, we have $w \approx_Q w'$. Question entailment goes from superquestions to subquestions; equivalently from more fine-grained partitions to less fine-grained partitions.

2.4.3 The Limiting Issue in a specific example

The Limiting Issue is a partition that could serve as the denotation for a natural language question. In the case of strict conditionals, the Limiting Issue depends on a modal base, so it is an essentially modal question. If we try to associate the Limiting Issue with a particular natural language question, which question it is will depend on the background analysis of modals.

To understand this better, let us consider a specific example. In (33), the exception-worlds are the worlds where A and B both went to the party. The Limiting Issue looks like (34).

(33) a. If A went to the party, then C went.

b. If A and B went to the party, then C didn't go.

(34) a. $L = \{\langle w, w' \rangle \mid w \in O \leftrightarrow w' \in O\}$

b. $O = \{w \in K \mid \exists w' \in D(w) \cap [\![\text{A went}]\!] : w' \in [\![\text{B went}]\!]\}$

(35) Possible paraphrases of the Limiting Issue:

a. Did B go in any of the worlds where A went?

b. If A went, is it possible that B went?

c. If A went, might B have gone?

These paraphrases capture the modal nature of the Limiting Issue. The Sufficient Truth theory predicts that (33) is infelicitous for any discourse participant who is considering the Limiting Issue (or a superquestion of it) prior to updating with the two conditionals in (33).

2.5 Sufficient Truth for Variably strict conditionals

So far we have seen limiting issues for the strict theory with sufficient truth. However, this is not the only possibility. It is also possible to combine the variably strict theory of conditionals with Sufficient Truth.

(36) Limiting Issues For the Variably Strict Sobel sequence

(i) ⌜if A, C⌝

(ii)⌜if A and B, not C⌝

Define the *Limiting Issue* for (i); (ii) to be $L = \{\langle w, w' \rangle \mid w \in O \leftrightarrow w' \in O\}$, where the set O is given by

$$O = \{w \in K \mid \exists w' \in f(w, A) : w' \in B\}$$

As before, if the QUD entails the Limiting Issue L, then conditional (i) cannot be true enough.[10] When the QUD entails the Limiting Issue, the Sobel sequence is bad. If it does not, the Sobel sequence can be saved. However, inattention to the Limiting Issue is not the only way for the Sobel sequence to sound felicitous. It might be the case that the evaluation world w *already* satisfies the constraint $f(w, A) \cap f(w, A \cap B) = \emptyset$. (This can only happen if $A \wedge B$ is taken to be false at w.)

When the QUD entails the Limiting Issue, the Sobel sequence is unacceptable. If it does not, the Sobel sequence can be saved. However, inattention to the Limiting Issue is not the only way for the Sobel sequence to sound felicitous. It might be the case that the evaluation world w *already* satisfies the constraint $f(w, A) \cap f(w, A \cap B) = \emptyset$. (This can only happen if $A \wedge B$ is taken to be false at w.) I will discuss this point further in Section 2.6.1. See Fig. 2.4 for detailed predictions of the account.

[10] A question P entails Q if whenever $w \approx_P w'$, we have $w \approx_Q w'$. Question entailment goes from superquestions to subquestions; equivalently from more fine-grained partitions to less fine-grained partitions.

$w \in A \wedge B$	$[\![(ii)]\!]^w = 0$	$[\![(ii)]\!]^w = \star$	$[\![(ii)]\!]^w = 1$
$[\![(i)]\!]^w = 0$	(impossible)	$0 \rightsquigarrow \#$	$0 \rightsquigarrow \#$
$[\![(i)]\!]^w = \star$	$0 \rightsquigarrow \#$	$\star \rightsquigarrow \checkmark$	$\star \rightsquigarrow \checkmark$
$[\![(i)]\!]^w = 1$	$0 \rightsquigarrow \#$	$\star \rightsquigarrow \checkmark$	(impossible)

Figure 2.4: Predictions
Predictions of the account in the problem case where $w \in A \wedge B$. In this case, w must be in both $f(w, A)$ and $f(w, A \cap B)$, so the domains overlap.

2.6 Reverse Sobel sequences

There is something odd about the variably strict theory with Sufficient Truth, which becomes clear once we look at the reverse Sobel sequences (henceforth reverse sequences). There are two ways to build felicitous Sobel sequences.

1. If $A \wedge B$ is known to be false, we can choose disjoint domains for (i) and (ii), making them both true.

2. Whether or not $A \wedge B$ is ruled out, we can make (i) and (ii) both true enough as long as the CI does not entail the LI.

There is a conceptual problem here. Part of the theory seems redundant. If $A \wedge B$ is known to be false, then we don't need non-maximality to derive a felicitous Sobel sequence, as long as we choose f to satisfy $f(w, A) \cap f(w, A \wedge B) = \emptyset$.[11] Moreover, we often don't know what kind of sequence we are looking at, which makes it hard to analyze particular cases.

There is also an empirical problem. Whenever the modal domains are disjoint, and the sequence is strictly true, the variably strict theory predicts that its reversal is felicitous.[12] Once we examine this prediction in more detail, we will look at some experimental data on the felicity

[11] This is reminiscent of the problem pointed out by Cross [2008], where relativizing the closest worlds to both the antecedent proposition *and* the similarity ordering results in a theory in which the similarity ordering barely makes a difference.

[12] This problem was anticipated by Križ (2015: 184). See Dohrn (2020: §4.2) for additional discussion.

of reverse sequences [Krassnig 2020], and see how we can approach it with a strict conditional analysis.

2.6.1 Issue-insensitive reverse sequences

On the sufficient truth theory, the explanation for infelicitous reverse sequences relies on a stipulation about how issues are raised. In the reverse sequence (ii) ⌜if A, C⌝; (i) ⌜if A and B, not C⌝, conditional (ii) raises an issue that makes (i) no longer true enough. The raised issue must entail the Limiting Issue.

But, on any variably strict semantics, Sobel sequences with disjoint domains can be strictly true. If (i) and (ii) are both strictly true, the QUD makes no difference for their felicity. So as long as $A \wedge B$ is epistemically impossible, we can choose some selection function that makes the reverse sequence felicitous, ignoring the QUD.

This odd feature of the variably strict Sufficient Truth theory appears to contradict experimental evidence on the subject. A study by Krassnig [2020] shows that reverse sequences with disjoint domains have intermediate acceptability. More precisely Krassnig shows that:

1. Reverse sequences with disjoint domains are significantly less acceptable than forward sequences.
2. However, reverse sequences with explicitly disjoint domains are significantly more acceptable than reverse sequences with overlapping domains.

The first prediction is not expected on the variably strict analysis, which predicts that reverse sequences with disjoint domains should behave like forward sequences.

Krassnig constructs reverse sequences with disjoint domains by explicitly ruling out $A \wedge B$ in the context. In (37) the possibility that Steve is wearing his helmet is excluded. Participants rated their acceptability on a 5-point Likert scale.

(37) Reverse sequence with disjoint domains Krassnig 2020: (10)

Context: Alex and her friend Steve enter a construction site. Steve doesn't wear his helmet, but carries it around in his hand. This annoys Alex, since it's a dangerous site.

Alex: If some construction material fell on your head right now and you wore a helmet, you would probably survive the incident; but if some construction material did fall on your head right now, you would certainly die. So, wear your goddamn helmet.

In (38), we have an example involving someone going onto a frozen lake. In this case, it is epistemically excluded that someone on the shore would stop the ice from breaking (since the interactants know that there will be nobody else there).

(38) Another example Krassnig 2020: (15)

(Said, over the telephone, to someone who is currently planning on going to a remote frozen lake next week that is known for usually nobody ever going there. It is known that said person has decided on going completely alone and is extremely adamant about it, because he wants get away from everything.)

Listen, if you walked on the thin ice next week while being supported by someone on the shore, the ice wouldn't break and you'd be fine; but if you DID walk on the thin ice next week, the ice would break and you would die! So, I'm begging you to be careful and not to go on the ice!

On the variably strict analysis, a reverse sequence is bad due to a shift in the QUD, whereby the Limiting Issue is raised by conditional (ii). However, when the domains are disjoint, no change in the QUD can make them infelicitous, so reverse sequences should sound like forward sequences. This is clearly not the case. Crucially, Krassnig's participants find examples like (37) to be significantly less acceptable than forward sequences, though they are much better than reverse sequences in which $A \wedge B$ is possible (where the domains are not disjoint).

2.6.2 Interim Conclusion on Reverse Sequences

There is limited experimental evidence on the acceptability of reverse Sobel sequences, and we should be cautious about interpreting the results of this single study. However, we can safely conclude that these results cast doubt on the variably strict theory for the following reasons. The referential strict theory for bare indicative conditionals, combined with Sufficient Truth, predicts that reverse sequences are infelicitious when $\ulcorner$if A and B, not $C\urcorner$ raises the limiting issue for $\ulcorner$if A, $C\urcorner$. The variably strict version does not make such clear predictions, because on the variably strict theory it is possible for the two conditionals $\ulcorner$if A and B, not $C\urcorner$ and $\ulcorner$if A, $C\urcorner$ to have disjoint domains. If the two conditionals have disjoint domains, then the sequence is predicted to felicitous no matter what happens with the QUD.

However we interpret Krassnig's other experimental results, it seems clear that reverse Sobel sequences are systematically degraded compared to forward Sobel sequences, and the variably strict theory, even when amended with Sufficient Truth, does not capture this. Considering that the strict theory is simpler, and the variably strict theory does not provide any added benefit for bare conditionals, these data appear to favor the strict theory.

It would be useful to replicate Krassnig's results and expand the data to include bare past and future indicative conditionals. A followup study would help to clarify the choice between strict and variably strict theories, once the additional mechanism of Sufficient Truth is added to the toolbox.

2.7 Comparison with other theories

In this section I compare my theory to relevant alternatives in the literature.

2.7.1 Moss [2012]

Moss [2012] is based on the idea that reverse Sobel sequences are not necessarily infelicitous. For Moss, when a speaker utters a counterfactual conditional (ii) if A and B, $\neg C$ makes salient a possibility that the speaker cannot rule out. The existence of this possiblity makes it infelicitous to follow up (ii) with (i) if A, C.[13]

The intuition here is quite similar to the one behind the Sufficient Truth account. However there are two important differences. First, salience is a primitive notion on Moss's theory. On the Sufficient Truth theory, we can think of a proposition p as salient just in case the QUD distinguishes p-worlds from $\neg p$-worlds. Second, and more importantly, Moss assumes that Sobel sequence conditionals are both true, and therefore runs into Willer's problem once again.

2.7.2 Klecha [2022]

Another relevant proposal is Klecha [2022], who separates Sobel sequences into two types: true Sobel sequences, which are consistent and freely reversible, and Lewis sequences, which are genuinely inconsistent. Klecha's point is that the sequences in the literature that are used to demonstrate the irreversibility of Sobel sequences are actually Lewis sequences, not true Sobel sequences. Lewis sequences are genuinely inconsistent, and their felicity is due to the fact that speakers can use conditionals imprecisely. The infelicity of reverse Sobel sequences is therefore a consequence of the general fact that it is easier to raise the standard of precision than it is to lower it.

Klecha's pragmatic theory of Lewis sequences is similar to both Moss's (2012) account and my own, in that it is a pragmatic account, not a semantic account. As such, it is not directly vulnerable to Willer's problem in the same way as variably strict semantic theories in the Lewis-Stalnaker tradition. Klecha [2022] does not offer an explicit theory of imprecision in conditionals,

[13] The exact possiblity that is raised depends on the background assumptions about counterfactual semantics. See Lewis (2018: 492-294) for a detailed discussion and critique.

but does convincingly argue in favor of a pragmatic approach based on imprecision. From that perspective, the contribution of this chapter can be seen as providing an additional argument for an imprecision account (using Willer's problem), as well as a concrete theory that explains *why* conditionals are imprecise. On the present account, conditionals are imprecise because *if*-clauses are plural definite descriptions of worlds that directly saturate the world argument of a predicate. Modal conditionals are quantificational structures, which is why Sobel sequences with modalized conditionals sound infelicitous. Thus, the present theory addresses a different piece of the puzzle than Klecha, but remains essentially compatible with his point of view.

2.7.3 Lewis [2018]

Karen Lewis [2018] gives a variably-strict account that is designed to deal with two kinds of data that have been problematic for previous accounts. The first is felicitous Heim sequences like (39-40), and the second is what she calls retraction sequences, like (41)

(39) a. If kangaroos had no tails and they used crutches, they would not topple over.

b. But if kangaroos had no tails, they would topple over. (Example adapted from Lewis 1973 (1,9).)

(40) a. (Holding up a dry match, with no water around:) If I had struck this match and it had been soaked, it would not have lit.

b. But if I had struck this match, it would have lit. (Example adapted from Stalnaker 1968 (106).)

Intuitively, Heim sequences (also known as reverse Sobel sequences) are felicitous when the first conditional antecedent is considered too remote a possibility to interfere with the truth of the second conditional.

In a retraction sequence, the first speaker appears to take back what they said, by using an epistemic possibility claim which appears to contradict the first conditional in the Sobel sequence.

(41) Sobel sequence with retraction Lewis 2018: 495-6

a. A: If Sophie had gone to the parade, she would have seen Pedro dance.

b. B: But of course, if Sophie had gone to the parade and been stuck behind someone tall, she wouldn't have seen Pedro dance.

c. A: Alright, I guess then, if Sophie had gone to the parade, she might not have seen Pedro dance.

Lewis argues that these two types of examples provide a dilemma for both the dynamic strict accounts and Moss's pragmatic theory. Dynamic strict theories handle retraction sequences well because the domain expansion that makes (41b) true also makes (41c) true. However, for dynamic strict accounts, domain expansion is a semantic operation, and domain contraction is not modeled. For this reason, they cannot countenance felicitous Heim sequences like (39-40). On the other hand, the pragmatic story found in Moss [2012] handles felicitous Heim sequences (which it was designed to do), but cannot render retraction sequences consistent.[14]

Lewis's own system makes each member true in its context by *reordering the closeness ordering on worlds* as new possibilities become relevant. Heim sequences are infelicitous when the first conditional introduces a relevant possibility. In this case, some $A \wedge B$-worlds can be close enough to falsify the second conditional if A, C. However, Heim sequences can be felicitous when $A \wedge B$ is deemed irrelevant, for example because it is too remote a possibility. Lewis's account also renders retraction sequences consistent. The second conditional in a Sobel sequence $A \wedge B > \neg C$ forces a reordering of worlds. After this reordering, some closest $A \wedge B$-worlds are among the closest A-worlds. It is precisely these $A \wedge B$ worlds that serve as witnesses for the *might*-claim.[15]

[14]This is only one of several arguments Lewis makes against Moss [2012]—see Lewis 2018: §3.

[15]In addition to providing an elegant twist on the variably strict semantics, Lewis significantly improves the

On my account, the ordering on worlds does not change as new possiblities become relevant. I allow the selection function f to stay constant through the discourse, unlike Lewis, who forces it to shift. Lewis's approach therefore relativizes the truth of a conditional to what discourse participants think is relevant. On my account, the literal truth of a conditional is not sensitive to relevance in this way. However, an indeterminate conditional can be *true enough* as long the worlds that make it indeterminate can be safely ignored. Lewis's approach is therefore vulnerable to Willer's problem, and is unable to make sense of Sobel-type counterexamples to Modus Ponens.

2.7.4 Ippolito [2020]

Ippolito [2020] has a recent theory of felicitous Heim sequences that aims to unify order asymmetries in Sobel sequences with Hurford disjunctions. The analysis relies on focus and Questions Under Discussion. In this respect, Ippolito's analysis shares certain features with the present one, particularly the idea that the felicity of Sobel sequences is sensitive to the QUD. However, like the other variably strict theories, this one is also bound to run into Willer's problem with epistemically possible antecedents and Modus Ponens.

2.7.5 Comparison of alternative accounts

I summarize some key features of these accounts, in comparison with my account, in the table below.

empirical breadth of the discourse on counterfactual Sobel sequences by showing that felicitous Heim sequences are easier to construct and more natural than had been previously assumed, and by introducing retraction sequences into the picture.

	Gillies [2007]	Moss [2012]	Lewis [2018]	This work
Bad reverse sequences	✓	✓	✓	✓
Good reverse sequences	*	✓	✓	✓
Retraction sequences	✓	*	✓	(✓)
Willer's problem	✓	*	*	✓

Figure 2.5: A comparison of theories of Sobel Sequences.

2.8 A compositional semantics for referential *if*-clauses

The pragmatic theory of Sobel sequences assumes that bare conditionals are trivalent. The trivalent semantics is required to feed the pragmatic derivations. In this section, I offer a concrete compositional semantics for conditionals that derives trivalent meanings for bare conditionals and bivalent meanings for modalized conditionals.

In the present system, all verbs take world arguments. I call type $\mathtt{st}$ terms *proposition radicals* to distinguish them from propositions (type $\mathtt{t}$). Example (42) contains two proposition radicals, $\ulcorner\mathsf{rain}\urcorner$ and $\ulcorner\mathsf{pour}\urcorner$. The *if*-clause has a silent domain restriction R, parallel to plural definites.

(42) If it rains, it pours. $\ulcorner\mathsf{if}_R\ \mathsf{rain},\ \mathsf{pour}\urcorner$

We first need to assign a syntax and a meaning to the *if*-clause. (43a) sets out the general syntactic structure for *if*-clauses, where terms α, β, and γ have type $\mathtt{st}$. The definition of $\ulcorner\mathsf{if}\urcorner$ as a λ-term is in (43b): $\ulcorner\mathsf{if}\urcorner$ takes a domain restriction R (a variable of type $\mathtt{st}$) and a proposition radical P (type $\mathtt{st}$) to a sum of worlds (type $\mathtt{s}$).[16] Function Application is notated by a dot, where $\alpha \cdot \beta$ simplifies to $\alpha(\beta)$ or $\beta(\alpha)$ depending on the types of the terms α and β (since at most one option will be type-correct).

(43) a. Syntax and Semantics of *if*-clauses $\ulcorner\mathsf{if}_\gamma\ \alpha, \beta\urcorner := ((\mathsf{if} \cdot \gamma) \cdot \alpha) \cdot \beta$

b. $\mathsf{if}_{(\mathtt{st})(\mathtt{st})\mathtt{s}} := \lambda R_{\mathtt{st}}.\lambda P_{\mathtt{st}}.\bigoplus(\lambda u_{\mathtt{s}}.R(u) \wedge P(u))$

[16]For convenience, the sum operator $\bigoplus$ is overloaded here as both a λ-term (type $(\mathtt{st})\mathtt{s}$) and a model-theoretic function mapping sets to sums.

In (44a), the syntactic rule applies so that ⌜pour⌝ takes the *if*-clause as an argument. In (44b), the denotation of the *if*-clause is derived, a plurality of type s. (As usual, $\mathfrak{D}_\tau$ is the domain of objects of type τ.) (44c) gives the semantics of the bare conditional.

(44) a. $\ulcorner \mathsf{if}_R\ \mathsf{rain},\ \mathsf{pour} \urcorner = ((\mathsf{if} \cdot R) \cdot \mathsf{rain}) \cdot \mathsf{pour} = \mathsf{pour}(\mathsf{if}(R)(\mathsf{rain}))$ (by def. 43a)

b. $[\![\mathsf{if}(R)(\mathsf{rain})]\!]^{\mathfrak{a}} = [\![\bigoplus(\lambda u_{\mathsf{s}}.R(u) \wedge \mathsf{rain}(u))]\!]^{\mathfrak{a}}$ (by def. 43b)

$= \bigoplus\{u \in \mathfrak{D}_{\mathsf{S}} \mid u \in \mathfrak{a}(R) \text{ and } u \in [\![\mathsf{rain}]\!]^{\mathfrak{a}}\}$

(*Paraphrase:* The worlds where it rains.)

c. $[\![\mathsf{if}_R\ \mathsf{rain},\ \mathsf{pour}]\!]^{\mathfrak{a}} = [\![\mathsf{pour}]\!]^{\mathfrak{a}}([\![\mathsf{if}(R)(\mathsf{rain})]\!]^{\mathfrak{a}})$ (apply 44a inside ($[\![\cdots]\!]^{\mathfrak{a}}$))

$= [\![\mathsf{pour}]\!]^{\mathfrak{a}}\Big(\bigoplus\{u \in \mathfrak{D}_{\mathsf{S}} \mid u \in \mathfrak{a}(R) \text{ and } u \in [\![\mathsf{rain}]\!]^{\mathfrak{a}}\}\Big)$ (apply 44b inside $(\cdots)$)

(*Paraphrase:* It pours in the worlds where it rains.)

By combining (44c) with the assumption that $[\![\mathsf{pour}]\!]^{\mathfrak{a}}$ obeys homogeneity, we get the full truth and falsity conditions for the bare conditional, shown in (45).

(45) $$[\![\mathsf{if}_R\ \mathsf{rain},\ \mathsf{pour}]\!]^{\mathfrak{a}} = \begin{cases} 1 & \text{if it pours in all } R\text{-worlds where it rains} \\ 0 & \text{if it pours in no } R\text{-worlds where it rains} \\ \star & \text{otherwise} \end{cases}$$

At this point, we should compare bare conditionals to modal conditionals like (46). While bare conditionals show homogeneity effects, modals remove homogeneity, just as quantifiers like *all* remove homogeneity for predicates and pluralities of individuals.

(46) If it rains, it must pour. $\ulcorner \mathsf{if}_R\ \mathsf{rain},\ \mathsf{must}_g\ \mathsf{pour} \urcorner$

I analyze modal conditionals along the same lines as Kratzer (1981: pp. 64-66). In her system, all modals are interpreted relative to a modal base and an ordering source. In (46), an overt *if*-clause serves to restrict the modal. I treat the *if*-clause as an argument to the modal in such cases, following Kratzer. The difference is that in my system, *if*-clauses denote sums of worlds rather than predicates of worlds. Because *if*-clauses denote sums, the modal quantifies distributively over the parts the sum.

I define ⌜must⌝ in (47a), and assign a syntactic structure to clauses headed by ⌜must⌝ in (47b).

(47) a. $\mathsf{must}_{(\mathsf{sst})(\mathsf{st})\mathsf{st}} := \lambda g_{\mathsf{sst}}.\lambda Q_{\mathsf{st}}.\lambda u_{\mathsf{s}}.\forall w.g(u)(w) \rightarrow Q(w)$

b. $\ulcorner \mathsf{must}_{\gamma}\ \alpha \urcorner := (\mathsf{must} \cdot \gamma) \cdot \alpha$

I implement the ordering source g as a designated variable (rather than a parameter of the interpretation function). The role of the modal base is played by the restriction R in the *if*-clause.[17]

The meaning of the modal conditional is derived as follows. First we simplify terms as in (48), applying the definitions introduced above.

(48) $\ulcorner \mathsf{if}_R\ \mathsf{rain},\ \mathsf{must}_g\ \mathsf{pour} \urcorner = ((\mathsf{if} \cdot R) \cdot \mathsf{rain}) \cdot ((\mathsf{must} \cdot g) \cdot \mathsf{pour})$ (by def. 47b)

must(g)(pour)(if(R)(rain)) = $\forall w.g[\mathsf{if}(R)(\mathsf{rain})](w) \rightarrow \mathsf{pour}(w)$ (by def. 47a)

Next, we consider the contribution of the ordering source g. As implemented here, g is a variable of type sst. It takes the plurality of worlds denoted by the *if*-clause, and returns a predicate that picks out the best ones. (Recall that $\leqslant$ means mereological parthood here.)

(49) $[\![g(u)(w)]\!]^{\mathfrak{a}} = \mathfrak{a}(g)(u)(w) = 1$ iff $w \leqslant u$ and, for all $w' \leqslant u :\ w \preceq_g w'$

(The predicate $\mathfrak{a}(g)(u)$ is true of all parts of u that are best according to g's ordering.)

The modal conditional quantifies over these best worlds, as under standard Kratzerian theories. Putting all of this together, we obtain the meaning of the modal conditional by first substituting in (50a) and giving truth conditions in (50b).

[17] When there is no *if*-clause, I assume the outer argument of ⌜must⌝ is saturated by a variable representing the modal base. I take this assumption to be in the spirit of Kratzer's analysis.

(50) a. $[\![\mathsf{if}_R \text{ rain, } \mathsf{must}_g \text{ pour}]\!]^{\mathfrak{a}} = [\![\forall w.g[\mathsf{if}(R)(\mathsf{rain})](w) \to \mathsf{pour}(w)]\!]^{\mathfrak{a}}$ (apply 48 in $[\![\cdots]\!]^{\mathfrak{a}}$)

b. $[\![\mathsf{if}_R \text{ rain, } \mathsf{must}_g \text{ pour}]\!]^{\mathfrak{a}} = 1$ iff every world w where $\mathfrak{a}(g)\Big(\bigoplus\{u \mid u \in \mathfrak{a}(R) \text{ and } u \in [\![\mathsf{rain}]\!]^{\mathfrak{a}}\}\Big)(w) = 1$ is such that $[\![\mathsf{pour}]\!]^{\mathfrak{a}}(w) = 1$

(Every g-best part of the plurality of worlds where it rains is a part where it pours.)

The only differences from the standard theory have to do with the *if*-clause, which has type `s` instead of `st`. When the world argument of a (homogeneous) verb is directly saturated by the *if* clause, the predicted truth and falsity conditions are exactly as they would be for a (homogeneous) verb that takes a plural definite description as an argument.

This is the last time I show such derivations in painstaking detail, since the compositional semantics is not the most interesting part of the analysis. Other implementations may be possible, but the important idea here is that ⌜if⌝ takes proposition radicals (type `st`) to pluralities of worlds (type `s`), so that *if*-clauses are referring expressions, parallel to plural definite descriptions.

2.8.1 Accounting for homogeneity

The scopelessness of conditionals with respect to negation is a consequence of homogeneity extended to worlds. This move was already anticipated by von Fintel [1997]. von Fintel implements homogeneity via a presupposition carried by definite descriptions and *if*-clauses. In the more general framework of Križ [2015], homogeneity is not a property of referential expressions, but of predicates.

This analysis requires a few modifications to the standard view of the lexicon. First, we allow lexical predicates to take pluralities of worlds as arguments [Schlenker 2006]. *If*-clauses can directly saturate these argument positions, or bind world pronouns from a distance. I assume most predicates are distributive and homogeneous with respect to worlds, meaning that a predicate of worlds S is evaluated at a plurality W as follows:

$$S(W) = \begin{cases} 1 & S(w) = 1 \text{ for all } w \in W \\ 0 & S(w) = 0 \text{ for all } w \in W \\ \star & \text{otherwise} \end{cases}$$

This assumption guarantees the following truth and falsity conditions for embedded conditionals, as desired:

$$[\![((\textsf{the} \cdot \textsf{book}) \cdot \textsf{burned}) \cdot (\textsf{if}_D \cdot (\textsf{it}_1 \cdot (\textsf{was dry})))]\!]^{\mathfrak{a}}(w)$$

$$= \begin{cases} 1 & \text{the book burned in every}_D\ w\text{-closest world where it was dry} \\ 0 & \text{the book burned in no}_D\ w\text{-closest world where it was dry} \\ \star & \text{otherwise} \end{cases}$$

$$[\![((\textsf{every} \cdot \textsf{book}) \cdot \textsf{burned}) \cdot (\textsf{if}_D \cdot (\textsf{it}_1 \cdot (\textsf{was dry})))]\!]^{\mathfrak{a}}(w)$$

$$= \begin{cases} 1 & \text{every book } x \text{ burned in every}_D\ w\text{-closest world where } x \text{ was dry} \\ 0 & \text{some book } x \text{ burned in no}_D\ w\text{-closest world where } x \text{ was dry} \\ \star & \text{otherwise} \end{cases}$$

$$[\![((\textsf{no} \cdot \textsf{book}) \cdot \textsf{burned}) \cdot (\textsf{if}_D \cdot (\textsf{it}_1 \cdot (\textsf{was dry})))]\!]^{\mathfrak{a}}(w) =$$

$$= \begin{cases} 1 & \text{no book } x \text{ burned in any}_D\ w\text{-closest world where it was dry} \\ 0 & \text{some book } x \text{ burned in every}_D\ w\text{-closest world where } x \text{ was dry} \\ \star & \text{otherwise} \end{cases}$$

To summarize, homogeneity immediately extends to pluralities of worlds. The trivalent meanings generated by homogeneous predicates effectively derive Conditional Excluded Middle for free. It is worth noting that this is not the only theory that delivers the same truth and falsity conditions for bare conditionals: for a supervaluationist approach, see Klinedinst [2011]. I leave a comparison of different possible trivalent theories to future work.

2.9 Domain expansion and dynamic strict conditionals

In dynamic strict theories of conditionals following , Sobel sequences are licensed via *domain expansion*. (Two examples of dynamic strict theories are von Fintel 1999 and Gillies 2004.) In these theories, epistemic conditionals are strict conditionals where the modal base is identified with the context set. Bare indicatives of the form ⌜if A, C⌝ carry a presupposition that there are some ⌜A⌝-worlds in the context set.

Consider the context of each conditional in (51) below. The first conditional (51a) is true only if there are no ⌜cat and landlord⌝-worlds in the context set. For this reason, when we evaluate the second conditional (51b), we get presupposition failure. To avoid presupposition failure, we expand the context set to include some ⌜cat and landlord⌝-worlds.

(51) a. If I get a cat, I'll be happy.
⌜if cat, happy⌝

b. If I get a cat and my landlord kicks me out, I won't be happy.
⌜if (cat and landlord), not happy⌝

Domain expansion theories say that we can add worlds to satisfy the presupposition of a conditional. But we can't subtract worlds to make a false conditional true. So forward Sobel sequences like (51) are ruled in, and reverse Sobel sequences are ruled out.

Nichols [2017] offers a persuasive and general critique of the domain expansion strategy. Nichols's first argument is that domain expansion incorrectly falsifies followup conditionals.

(52) a. If Lars had come to the party, it would've been fun.
⌜if party, fun⌝

b. If he hadn't been at a wedding on the West Coast that day, he would've gone to the beach.
⌜if (not wedding), beach⌝

The sequence in (52) is intuitively consistent, but predicted to be inconsistent in domain-expansion theories. The reasoning is as follows: Notice that ⌜wedding⌝ and ⌜party⌝ are assumed to be mutually exclusive, as are ⌜beach⌝ and ⌜party⌝. Crucially, this means that all ⌜party⌝-worlds are both ⌜not wedding⌝-worlds and ⌜not beach⌝-worlds.

Domain expansion after (52a) brings in ⌜party⌝-worlds. But then, the truth of (52b) requires that all ⌜not wedding⌝-worlds are ⌜beach⌝-worlds, which is falsified by the existence of ⌜party⌝-worlds.

Nichols's second argument is that contraction re-licenses infelicitous reverse Sobel sequences. If we allow domain contraction to save sequences like (52), then the contracted domain would license incoherent reverse Sobel sequences. (53) provides an example of this.

(53) *Context: I only have time to care for one pet.*

a. If I get a cat and my landlord kicks me out, I won't be happy.
⌜if (cat and landlord), not happy⌝

b. If I don't get a dog, I'll get a parrot.

⌜if (not dog), parrot⌝

Domain contraction: Remove ⌜cat and landlord⌝-worlds.

c. So, if I get a cat, I'll be happy.

⌜if cat, happy⌝

Domain expansion: Introduce new ⌜cat and (not landlord)⌝-worlds.

(53c) is just as bad with (53b) as it is without it. That is, removing (53b) from the sequence of conditionals does not change the status of (53c).

But if (53b) were licensed by domain contraction, we would expect it to improve (53c), because the contracted domain should license it. If domain contraction is not allowed after updating with (53b), then the worlds where the speaker gets a cat and their landlord kicks them out will be enough to falsify (53c). The domain expansion theorist could try to escape this dilemma by disallowing domain expansion after (53c). This could be done by introducing a further constraint on domain expansion. However, it is not clear what form that constraint would take, and repairing domain expansion is outside the scope of this work.

In conclusion, while it may be possible to fix the outstanding technical issues with dynamic domain expansion theories, I feel that these problems provide enough motivation to pursue pragmatic approaches to Sobel sequences.

2.10 Conclusion

In this chapter, I have offered a formalization of Križ's plural, referential theory of conditionals, and offered a novel argument in its favor. The theory derives the differences between bare conditionals and quantified conditionals with respect to homogeneity, including data in which conditionals are embedded under negative quantifiers.

Most interestingly, I have demonstrated that the non-maximality approach to Sobel sequences (a consequence of the referential analysis) has an unforeseen advantage over the Lewis-Stalnaker-

Kratzer approach to Sobel sequences, in that it avoids certain bad entailments that the latter theory predicts. I have shown further that the plural, referential analysis can be cashed out in both strict and variably strict versions, and that these two versions are interestingly different from each other.

In particular, the variably strict version seems to predict that reverse Sobel sequences should sound better than they actually do. The strict version's predictions are more straightforward: When the conditional ⌜if A and B, not C⌝ raises the Limiting Issue for the conditional ⌜if A, C⌝, the latter will be infelicitious. I consider this to be a point in favor of the strict theory.

In this work I have focused on indicative Sobel sequences, since indicative conditionals were what motivated Willer's problem. It is possible that the strict theory fits indicative conditionals best, while subjunctive conditionals should be treated as variably strict. The argument against the variably strict theory—Willer's problem—applies to all epistemically open conditionals, which are usually indicative. In future work on this topic, it may be profitable to carefully distinguish between counterfactual subjunctive conditionals and epistemically open subjunctive conditionals in Sobel sequences. This is not easy, since non-counterfactual subjunctive conditionals are less natural than counterfactuals, and this may make the Sobel sequence data harder to assess. Nonetheless, this would be a natural place to look.

3 | A Referential, Plural Account of Habituals and Temporal Adverbs

3.1 Introduction

Negated habitual sentences such as (54a) have a negative universal reading. The first clause of (54a) seems to entail that Connie *never* calls her mother on a Saturday. This explains the infelicity (#) of the second clause, which contradicts the negative universal entailment. Overt quantificational adverbs such as *always* in (54b) and *every Saturday* in (54c) render the followup acceptable.

(54) a. Connie doesn't call her mother on Saturday, # only every other Saturday.

b. Connie doesn't **always** call her mother on Saturday, ✓ only every other Saturday.

c. Connie doesn't call her mother **every** Saturday, ✓ only every other Saturday.

This pattern resembles the well-known *homogeneity effects* observed with plural definites. While (55a) only has a negative universal reading, (55b) has a global non-universal reading (easily explained by low scope of *all* under negation).

(55) a. I didn't eat the cupcakes, # but I ate half of them.

b. I didn't eat **all** the cupcakes, ✓ but I ate half of them.

Homogeneity effects were first analyzed by Fodor [1970], and Löbner [1985], who assume that plural definite descriptions carry a homogeneity presupposition. These analyses put homogeneity on a par with the gaps that result from presupposition failure [von Fintel 1997; Gajewski 2005; Ferreira 2005]. However, a more recent wave of analyses [Malamud 2012; Magri 2014; Križ 2015] have attempted to relate homogeneity effects to exception-tolerance, significantly expanding the empirical scope and generality of the phenomenon.

I propose that Križ's (2015) re-framing of the problem of homogeneity, and in particular the link between homogeneity and non-maximality, paves the way for an analysis of habitual readings that does not rely on any specialized silent aspectual operators. On this account, the properties of habitual sentences like (54a) arise from the interaction of independently-needed mechanisms, and not from the presence of a silent quantifier.

3.1.1 Homogeneity

I assume that all predicates of times are *homogeneous* in the sense of Križ [2015]. Križ initially applies homogeneity to predicates of individuals: A plurality in the extension of a predicate $\lambda x.P(x)$ must not overlap with any plurality in $\lambda x.\neg P(x)$. Thus, (56a) means that all of the windows are open, and (56b) means that none of the windows are open.[1]

(56) a. The windows are open. $*\mathsf{open}(\bigoplus \mathsf{window})$

b. The windows are not open. $\neg * \mathsf{open}(\bigoplus \mathsf{window})$

The extension to predicates of times is entirely natural: When a sentence S is true of some plurality of time intervals t, it cannot be false of any plurality s that overlaps with t. From this, it follows that (57a) means that Riley swims on every morning, and (57b) means that Riley doesn't swim on any morning.

[1]In (56) and (57), I do not indicate the contextual domain restrictions, but I assume that the domain restrictions are identical across the (a) and (b) examples. See Križ (2015: 74-45) for discussion on domain restrictions. The * operator closes the extension of a predicate under sums.

(57) a. Riley swims in the morning. $*\text{swim}(\bigoplus \text{morning})(\text{riley})$

b. Riley doesn't swim in the morning. $\neg * \text{swim}(\bigoplus \text{morning})(\text{riley})$

Homogeneity effects are observed in both unrestricted temporal anaphora as in (58) and explicit restriction by frame adverbials as in (57). Here, t is a free variable that points to a salient time interval.

(58) a. Riley swims. $*\text{swim}(t)(\text{riley})$

b. Riley doesn't swim. $\neg * \text{swim}(t)(\text{riley})$

I assume the domain of time intervals is nonatomic [von Stechow 2009]: Every time interval is a sum of time intervals, so homogeneity can be applied to all predicates of times, regardless of the properties of their temporal arguments. Time pronouns are never restricted to atomic referents. Thus, any tensed sentence S potentially shows homogeneity effects, since time intervals in the extension of S overlap with intervals that are outside the extension of S.

3.1.2 Non-maximality

The theory that habituals involve plural reference also explains their exception-tolerance. Malamud [2012] and Križ [2015] analyze examples in which speakers produce and accept plural predications of the form $P(x)$, even in contexts where only some parts of the plurality x satisfy the predicate P. This phenomenon is called *non-maximality*. For example, consider the sentence (59) in two different contexts.

(59) Riley drinks coffee in the morning.

(60) Question Under Discussion *favors* a non-maximal reading

Context: Riley drinks coffee only a few mornings a week, but never in the afternoon.

Question Under Discussion: *Does Riley drink coffee in the afternoon?*

Riley drinks coffee in the morning.

(61) Question Under Discussion *blocks* a non-maximal reading

Context: Riley drinks coffee only a few mornings a week, but never in the afternoon.

Question Under Discussion: *Does Riley drink coffee every day?*

Riley drinks coffee in the morning.

The non-maximal use is in (60), where the exact proportion of mornings where Riley drinks coffee is not relevant. In (61), where it is relevant, the habitual is no longer felicitous. Analyses of habituals that stipulate a silent adverbial quantifier do not explain temporal homogeneity and non-maximality.[2]

3.1.3 The proposal

The idea that habitual sentences involve plural reference and homogeneity effects is due to Ferreira (2005: 81-90). Since Ferreira's original proposal, Križ [2015] has provided the field with a general theory of homogeneity effects. Križ's theory has two properties that are crucial for analyzing habitual sentences. First, it is not specific to either definite descriptions or individuals, allowing a natural extension to time pronouns and temporal adverbials. Second, it predicts that homogeneity and exception tolerance are linked, via the theory of non-maximality. This second property is most important for capturing the novel data in Section 3.3.

The theory is based on the following informal generalization, to be formalized later on. Note that in the present work I use the term **indeterminate** instead of undefined, to keep homogeneity effects separate from presupposition failure.

(62) **Homogeneity Generalization** Križ 2015: 7

No individual in the positive extension of a [homogeneous] predicate [can] overlap with an individual in its negative extension.

[2] See Deo [2009] for other arguments against Q-adverb approaches to habituals.

The term *individual* above is intended to apply to pluralities as well as atomic individuals. Consider the sentence below, which shows the application of a homogenous predicate to a plural argument, resulting in an indeterminate truth value.[3] Throughout the chapter, I use # to indicate infelicity in a given context.

(63) QUD: How many people jumped in the lake?
Context: Six boys are playing by the lake. Five of them jump in.
The boys jumped in.

If it is understood that *the boys* refers to the six boys in our scenario, then we predict that the truth value of (63) will be indeterminate. This is because the set of five boys who jumped overlaps with the denotation of *the boys*, but is not equal to it.

3.1.4 Roadmap

The structure of the chapter is as follows. In Section 3.2, I introduce the systematic parallels between definite plurals and habitual sentences. In Section 3.3, I provide novel arguments that exception-tolerance in habituals depends on the Question Under Discussion (QUD) [Roberts 2012]. Thus, Križ's QUD-based analysis of non-maximality is the theory that is best equipped to deal with exception-tolerance in the temporal domain. In Section 3.4 I compare the present approach to previous attempts to derive the exception tolerance of habituals. In Section 3.5 I conclude.

In the Appendix, I present a compositional semantics for tense and temporal adverbials that derives the trivalent readings required for the theory of non-maximality. While it is fully formal and useful for understanding the proposal, the ideas in the compositional fragment are based

[3] Löbner (1985: 286:(12)) also notes that predication involving non-plural individuals requires homogeneity once we consider their parts. For example, he observes that *John is dirty.* is true if John is totally dirty, false if John is totally clean, but intermediate if John is only partly dirty

heavily on the trivalent type theory apparatus in Križ (2015: Chapter 2), and are not crucial for the novel empirical contributions in Sections 3.2 and 3.3.

3.2 Key data: bare habituals

In this section, I compare quantified habitual sentences with **bare habituals**—habituals without overt quantifiers. While quantified habituals are ambiguous under negation, bare habituals are not. Following Ferreira [2005], I argue that the lack of ambiguity in negated bare habituals is due to a homogeneity effect, parallel to those found with plural definite descriptions of individuals under negation.

In Section 3.2.1, I distinguish between bare habituals, which display homogeneity effects, and quantified habituals, which do not. In Section 3.2.2, I present novel data and use them to argue that habitual sentences show homogeneity effects, and that these homogeneity effects are entirely parallel to those that Križ observes with plural definites.

3.2.1 Classes of habitual sentences

Habitual sentences can be divided into two broad classes: bare habituals like (64a) and sentences involving adverbial quantifiers like (64b). I adopt the terminology *bare* to mean *non-quantificational*, following Ferreira [2005].

(64) a. Semantics Group meets (on) Friday mornings. (bare habitual)

b. Semantics Group meets (on) every Friday morning. (quantified habitual)

One might object that *(on) Friday mornings* functions as an adverbial quantifier, but when we add negation the two examples come apart, as we can see in (65) and (54).[4]

[4] As before, # indicates infelicity in context, but in (65) and (54) there will be infelicity in *any* context, due to the contradiction.

(65) a. SG doesn't meet on Friday mornings, # only every other Friday.

b. SG doesn't meet every Friday morning, only every other Friday.

These facts are not just a consequence of the explicit temporal modifiers like *on Friday mornings*. It turns out that nothing changes when we consider bare habituals with no temporal modifiers. The pattern of judgments that we observe with the temporal PP *on Friday mornings* in (64a) is exactly the same as the pattern in (66).

(66) a. #Anya doesn't swim, but she does sometimes.

b. Anya doesn't always swim, but she does sometimes.

Quantificational adverbials (Q-adverbs) like *always* and *every Friday*, on the other hand, produce scope ambiguities under negation.[5]

Once these facts are considered together, we can safely separate out at least two kinds of temporal modifiers: those that, like quantifiers, enter into scopal ambiguities with negation, and those that do not.

[5] Sentence-final Q-adverbs can be scopally ambiguous, as in (67a). When Q-adverbs are topicalized, they generally take wide scope, as in (67b).

(67) a. Ben doesn't swim every morning. But he does swim some mornings.

b. Every morning, Ben doesn't swim. # But he does swim some mornings.

In contrast, bare habituals with non-quantificational temporal adverbials have the same reading whether the adverbial is topicalized or not, though there may be some information-structural differences. Neither (68a) nor (68b) can have a narrow scope universal reading.

(68) a. Ben doesn't swim when it's morning. # But he does swim some mornings.

b. When it's morning, Ben doesn't swim. # But he does swim some mornings.

3.2.2 Homogeneity properties of habituals

In this section, I look at several patterns of judgments that function as diagnostics for homogeneity. In each case, the key patterns that Križ [2015] found for plural definites can be replicated for habitual sentences.

3.2.2.1 The *well*-test

There are certain situations in which it is not appropriate to either affirm or deny a habitual sentence. According to the homogeneity theory, this is because the habitual sentence is neither true nor false in the context. For plural definites, Križ uses responses with *well* as a diagnostic for indeterminate truth values. I adapt this test to bare habituals in (69). To conclude that sentence (69A) has an indeterminate truth value, the *well*-response must be not only available, but preferred to a *no*-response. The mere availability of a *well*-response is not sufficient to establish that the sentence being responded to is indeterminate.

(69) *Context: Albert has a habit of running in the morning, especially when the weather is good. Today, he had an early meeting, so he didn't make it.*

A: When it's sunny, Albert runs in the morning.

B: Well, he didn't today.

B': ?? No, he didn't today.

(70) *Context: Annie, Bonnie, and Connie are occasionally late to school. Bonnie's attendance is the best, but even she comes late sometimes.*

Annie: Bonnie comes to school on time.

Connie: Well, most of the time.

Connie': ?? No, most of the time.

Plural definites behave the same way. When the plurality denoted by the definite is not in the extension of the predicate, but overlaps with a plurality that is in the extension, the *well*-response is preferred.

(71) *Context: Half of the professors smiled.* Križ 2015: 75:(14)

A: The professors smiled.

B: Well, half of them.

B”: ?? No, half of them.

(72) *Context: In a large graduating class, most of the kids join together to sing a song.* (my example)

A: The kids sang.

B: Well, most of them did.

B’: ?? No, most of them did.

While the *no*-responses in all these examples are dispreferred, they are not impossible. I suggest that this variability in judgments results from subtle shifts in the Question Under Discussion [Roberts 2012]. As discourse participants shape the flow of information according to their conversational goals, they constantly re-negotiate the QUD using both explicit and implicit means. This fluctuation in the QUD means that *no*-responses occasionally show up as responses to indeterminate sentences. In those cases, they act as a signal that speakers intend to make finer distinctions than are relevant to the current QUD. I return to this subject in Section 3.3, where I discuss the role of the QUD and its relation to exception-tolerance.

3.2.2.2 *All*, *always*, and distributive quantifiers

Adding *always* or a distributive quantifier over times removes homogeneity from habitual sentences. To see this, compare the negated bare habituals in (73) with the negated quantified habituals in (74) and (75). The quantified sentences have far weaker truth conditions than (73). In fact, they typically come with implicatures that there are relevant occasions where Ben *does* bite his fingernails. In other words (74) and (75) are not only weaker, but usually implicate that (73) is not true.

(73) Ben doesn't bite his fingernails.
⇒ Ben never bites his fingernails.

(74) Ben doesn't always bite his fingernails.
⇒ At some of the relevant times, Ben does not bite his fingernails.

(75) Ben doesn't bite his fingernails every day.
⇒ Some days, Ben does not bite his fingernails.

Križ notices exactly the same pattern when examples with plural definites are compared to examples with *all* or distributive quantifiers over individuals.

(76) The kids didn't sing. ⇒ None of the kids sang.

(77) All the kids didn't sing. ⇒ Not every kid sang.

Sentence (77) has a reading on which it is possible that some but not all of the kids sang, unlike (76).

3.2.2.3 Unmentionability of exceptions

Habituals are known to be exception-tolerant in certain contexts [Carlson 2012; Deo 2009]. This mirrors the exception tolerance of plural definites. The full theory behind this is explained in Section 3.3. For now, I note that even when exceptions are possible, it is infelicitious to mention them in a followup.

(78) *Unmentionability of exceptions in habituals*

a. # Ben doesn't bite his fingernails, he only does it once a month.

b. # Ben doesn't bite his fingernails, he only does it after stressful meetings.

This is exactly parallel to the situation with plural definites, where exceptions are occasionally tolerated, but not mentioned explicitly without further explanation.

(79) *Unmentionability of exceptions in plural definites* Kroch 1974: 191:(5a,7a)

a. # Although the men in this room are angry, one of them isn't.

b. # Although the Jones's horses died in the barn fire, some of them didn't.

The source of this restriction is that whether a habitual tolerates exceptions depends on the relevance of those exceptions in the given context. In particular, the Question Under Discussion might draw a sharp boundary between mixed scenarios and homogeneous scenarios (in which case exceptions are not tolerated) or it might group some mixed scenarios together with some homogeneous scenarios. In the second case, those exceptions are irrelevant to the QUD, so mentioning them explicitly as in (79) would violate the Maxim of Relevance. We will return to these points in detail.

In summary, plural definites and habituals pattern together with respect to three diagnostics: Strong readings under negation, the *well*-test, and the unmentionability of exceptions.

3.3 Non-maximality in habituals

In this section, I present a new generalization about habitual sentences, namely that they tolerate exceptions only when those exceptions do not matter for resolving the QUD. Previous accounts of the exception-tolerance of habituals do not account for this fact, as I discuss in Section 3.4.

I pursue an analysis on which the exception-tolerance of habituals is a kind of non-maximality. According to the theory laid out in Križ [2015], in any area of the grammar in which we observe homogeneity effects, we should also expect exception-tolerance. This is because sentences with an indeterminate truth value can still be accepted as *true enough* under certain circumstances (to be defined shortly).

According to the theory of non-maximality, speakers are not required to say only sentences which they believe to be true, as in Grice's (1975) Maxim of Quality. Rather, they have a weaker responsibility to only say what they believe is true enough in the current context.

(80) **(Weak) Maxim of Quality** Križ [2016]

Say only sentences which you believe to be true enough.

Informally, a sentence is *true enough* if the worlds in which it is true are not distinguished from the evaluation world by the Question Under Discussion.[6] This will be fully formalized in (91) below.

3.3.1 Non-maximality with plural definites

Consider the indeterminate sentence in (81). Križ observes that this sentence is accepted as true enough in a mixed context as long as the exceptions do not matter for the Question Under Discussion.

(81) Non-maximal plural definites Križ 2015: 73

Context: Professor Smith never smiles after talks. After Sue's talk, every professor smiled but Smith, who wore a neutral expression.

The professors smiled. [$\star \rightsquigarrow 1$]

[6] Križ uses the term *Current Issue* rather than *Question Under Discussion* to name this contextual parameter, which is formally a partition over the set of possible worlds. There are distinctions between the two, but I use the QUD here for the sake of clarity. See Section 1.3 for further discussion and references.

In this case, we can take the Question Under Discussion to be something like (82).

(82) Was Sue's talk well-received?

If we know that Smith never smiles after talks, then we might think that Smith doesn't smile after even the very best talks. Thus, the world where every professor but Smith smiles will still count as a world where Sue's talk is well received. The Question Under Discussion will not distinguish this mixed world from the homogeneous worlds where all the professors smile.

Varying the QUD changes the acceptability of responses to plural definites. For example, in (83), the QUD is whether the discourse participants are being too loud. In this case, the fact that some of the townspeople might be awake at 2AM is not relevant to the QUD. What matters is that making noise is unacceptable as long as *enough* people are asleep.

(83) *Context: It's 2AM in a small town.*

A: Don't make noise, the townspeople are asleep! [$\star \rightsquigarrow 1$]

B_{well}: Well, we're awake. [1]

B_{no}: #No, we're awake. [$\star \rightsquigarrow 0$]

Because A's assertion is true enough, the denial in B_{no} is false enough, violating the Weak Maxim of Quality (80).

We can contrast this with an example based on Lasersohn (1999: 523) cited by Križ (2015: 72-73). In this example, the Question Under Discussion is whether the experiment can begin, so there will be no mixed worlds in the same cell as the homogeneous world where every participant is asleep. Thus, A's assertion is false enough, and the denial is true enough.

(84) *Context: A sleep study. The study can only begin once the participants are asleep. One person is still tossing and turning.*

A: #The participants are asleep. [$\star \rightsquigarrow 0$]

B_{well}: Well, one person is still awake. [1]

B_{no}: No, one person is still awake. [$\star \rightsquigarrow 1$]

As we will see, habitual sentences are sensitive to the Question Under Discussion in precisely the same way.

3.3.2 Non-maximality in habituals

In this section I present examples of habitual sentences in contexts where exceptions are tolerated. In each pair of examples, the Question Under Discussion is manipulated in various ways, and it turns out that the interpretation of the habitual is exception-tolerant whenever the QUD is not sensitive to small exceptions.

In the first example in (85), Annie's assertion is perfectly natural. Intuitively, this is because the discourse participants' attendance is very bad, and they are comparing themselves to Bonnie, whose attendance is quite a bit better.

(85) *Context: Annie and Connie are late to school almost every day, but Bonnie's attendance is generally good. Bonnie comes to school on time about on most days, but a few times a month she is late. Annie says to Connie:*

Annie: Bonnie comes to school on time. [$\star \rightsquigarrow 1$]

In this context, I assume that the Question Under Discussion is (86). Recall that Annie's assertion is strictly true only in the worlds where Bonnie is always on time. What is crucial about (86) is that the worlds where Bonnie is always on time are in the same alternative as the actual world, where Bonnie's attendance is imperfect, but still generally good.

(86) Whose attendance is generally good?

(Contains the alternative: Bonnie's attendance is generally good.)

Further evidence is available when we compare responses with *well* to denials with *no*. When we compare Connie's *well*-response to the *no*-denial in (87), the denial is degraded.[7]

(87) *Context: Same as (85).*

Annie: Bonnie comes to school on time. [$\star \rightsquigarrow 1$]

Connie: Well, she does most of the time. [1]

Connie': ??No, but she does most of the time. [$\star \rightsquigarrow 0$]

When we impose a more stringent QUD, as in (88), small exceptions (occasional lateness) become important. In (88), all attendance is being logged on a regular basis. In this kind of context, the *no*-response is felicitous.

(88) *Context: Stickers are being given out for perfect attendance. Bonnie comes to school on time about on most days, but a few times a month she is late.*

Annie: Bonnie comes to school on time. [$\star \rightsquigarrow 0$]

Connie: No, but she does most of the time. [$\star \rightsquigarrow 1$]

In this case, the Question Under Discussion is (89). Connie's denial is true enough in this context because the mixed worlds (where Bonnie's attendance is imperfect) are still worlds where she does not get a sticker, just like the worlds where Bonnie never comes to school on time.

(89) Who gets a sticker?

(Contains the alternative: Bonnie gets a sticker.)

Note that the *well*-response is also felicitous in the strict context. This is expected given the parallel facts with non-maximality in plural definites—recall the sleep study scenario in (84).

[7] The reason why the *no*-response is degraded, but not altogether impossible, is that in real discourse, there is an outside chance that the Question Under Discussion can shift as discourse participants renegotiate their conversational goals. At any point in time, one or both of the discourse participants could decide to impose a more exacting QUD, according to which the worlds where Bonnie is usually on time and the worlds where Bonnie is always on time will not occupy the same cell. The availability of silent QUD-shifting is a challenge for all theories that crucially rely on the QUD, and a full resolution of this problem is outside the scope of this study.

For the last example, let us consider a short dialogue in which discourse participants have different views on the Question Under Discussion, and produce apparently incompatible habitual sentences based on their divergent views. In (90), the Question Under Discussion is whether Connie is healthy. By committing to the proposition *Connie smokes* when in fact Connie only smokes very rarely, Annie reveals that her version of the Question Under Discussion divides worlds in which Connie never smokes from the actual world, in which she smokes rarely. Bonnie's response reveals that her version of the Question Under Discussion groups these worlds together.[8]

(90) *Context: Bonnie is asking Annie about Connie's health. Bonnie thinks occasional smoking is not a significant health issue, but Annie thinks that it is.*

Annie: Connie smokes.

Bonnie: How often?

Annie: Well, only once a year, at New Years.

Bonnie: Oh, so she doesn't smoke then.

Interestingly, these are two different versions of the same Question Under Discussion (*Is Connie healthy?*), but speakers behave differently depending what they think the actual content of the issue is. The result is a dialogue that is entirely plausible, but difficult to explain unless bare habitual sentences are sensitive to subtle shifts in the Question Under Discussion.

3.3.3 Analysis of non-maximality

I assume a trivalent semantics and adopt Križ's notation for the positive extension $[\![S]\!]^+$ (the set of all worlds that make S true) and the negative extension $[\![S]\!]^-$ (the set of all worlds that make S false). The worlds at which S is indeterminate will be in neither set. As we stated before, a

[8] One might object that this example shows that *smokes* is a vague predicate. I address this objection is Section 3.4.

sentence whose truth value is indeterminate may still be *true enough* for the purposes of the conversation. Here, we make this notion formally precise.

(91) **Sufficient Truth** Križ [2016]

We write $\simeq_I$ for the equivalence relation that holds of two worlds u, v iff u and v are in the same cell of an issue I. A sentence S is **true enough** in world w with respect to I iff there is some world w' such that $w' \in [\![S]\!]^+$ (S is true in w') and $w \simeq_I w'$.

In addition, discourse participants make their utterances relevant to the discussion by *addressing the (Current) Issue.*

(92) **Addressing an Issue** Križ [2016]

A sentence S may be used to address an issue I only if there is no cell $i \in I$ such that i overlaps with both the positive and the negative extension of S, i.e. S is true in some worlds in i and false in others.

In other words, no possible answer to I may include both worlds where S is true and worlds where S is false.

Consider the sentence (93). In the Appendix, I give a compositional semantics that derives logical translations for these sentences with the desired truth conditions. Here, I give informal paraphrases to simplify the presentation.

(93) On school days, Bonnie comes in on time.

$[\![(93)]\!]^+ = \{w \in D_s \mid$ Bonnie is on time on *all* school days in w

$[\![(93)]\!]^- = \{w \in D_s \mid$ Bonnie is on time on *no* school days in w

Suppose w^1 is a world where Bonnie is unfailingly on time, w^0 is a world where Bonnie is unfailingly late, and $w^\star$ is a world where Bonnie is mostly on time, but occasionally late. The world w^1 will be in the positive extension $[\![(93)]\!]^+$, w^0 will be in the negative extension $[\![(93)]\!]^-$, and $w^\star$ will be in neither.

Consider the two possible Question Under Discussions below. Since they are polar questions, each Issue I is modeled as a set of two cells, i_1 (the positive answer) and i_0 (the negative answer). The lax Question Under Discussion I^{lax} (94a) is such that the positive answer contains both w^1 and $w^{\star}$, while for the strict Question Under Discussion I^{strict}, the positive answer only contains w^1 (out of the three worlds considered).

(94) a. Is Bonnie generally on time?

$I^{lax} = \{i_1^{lax}, i_0^{lax}\}$ $\quad$ $w^{\star}, w^1 \in i_1^{lax}$ $\quad$ $w^0 \in i_0^{lax}$

b. Does Bonnie get a sticker for perfect attendance?

$I^{strict} = \{i_1^{strict}, i_0^{strict}\}$ $\quad$ $w^1 \in i_1^{strict}$ $\quad$ $w^{\star}, w^0 \in i_0^{strict}$

Whenever the actual world is a mixed world like $w^{\star}$, I predict that the habitual sentence (93) will be true enough when the Question Under Discussion is lax (94a), and will be false enough when the Question Under Discussion is strict (94b). This is precisely the situation in examples (85-88) from the previous section, where the context ensures that the actual world is a mixed world.

3.4 Comparison to previous work

The idea that bare habituals involve plural predication of events has been around since at least Ferreira [2005], but the present work gives the first account of exception-tolerance in habitual sentences that captures its context-sensitivity. The non-maximality account has the distinct advantage that its predictions can be tested by manipulating the Question Under Discussion. What we find is that bare habituals are not exception-tolerant across the board, but only when their exceptions are not relevant to resolving the Question Under Discussion.

In this section I discuss previous approaches to exception tolerance in habitual sentences. In many cases, authors have noted that habituals tolerate exceptions, but none have systematically connected these exceptions to particular contextual parameters such as the Question Under Discussion. As a result, they make no predictions about contexts in which exceptions matter, which I have called the strict contexts.

3.4.1 Ferreira [2005]

Ferreira [2005] first proposed that bare habituals should be analyzed using plural event predication. He uses a version of event semantics in which events are atomic. This allows him to analyze *when*-clauses in sentences like (95) using the semantics in (96).

(95) When John writes a romantic song, he goes to the Irish pub. Ferreira 2005: 63:(87)

The meaning for *when John writes a song* is true of those pluralities whose proper parts satisfy the event description $\lambda e.\exists y[\text{song}(y) \wedge \text{write}(e, j, y)]$.

(96) Ferreira's analysis of distributive *when*-clauses

$[\![\text{when John writes a song}]\!] = \lambda E.\forall e[e < E \rightarrow \exists y[\text{song}(y) \wedge \text{write}(e, j, y)]]$

(True of a plurality of events E if every proper part of e is a John-writes-a-song event.)

Ferreira then applies a definite determiner to (96) before composing it with the main clause. The result is that (95) is true if the unique plurality of events whose proper parts are songwriting-events is plurality of pub-going events. If such a definite plurality exists, this gloss does not seem to tolerate any exceptions.

The main difference between the present work and Ferreira's analysis is that Ferreira focuses on the modal properties of habitual sentences, which he sees as parallel to the modal properties of the progressive (Dowty 1979, Landman 1992, Portner 1998). One of Ferreira's primary goals is to account for the common modal imperfective core of habitual and progressive readings across languages. Ferreira (2005: 57-59) suggests that the exception-tolerance of habitual sentences can be explained via the modal semantics, but once the modal semantics is introduced in Chapter 4 of that dissertation, there is no explicit discussion of exceptions. Thus, Ferreira's predictions about exceptions are not clear, and to the extent that exceptions can be accommodated, there is no expectation that they should depend on the Question Under Discussion.

3.4.2 Deo [2009]

Deo [2009], like Ferreira, aims to account for the shared modal properties of imperfective verbs, whether read progressively or habitually. Deo (2009: 483-484) argues against an event-quantification analysis, observing that even explicitly domain-restricted habituals like (95) are exception-tolerant. She concludes that a quantificational account cannot easily build in exception-tolerance via implicit quantifier domain restriction.

Deo's habitual semantics has two components. First, she implements a Dowty-style modal semantics using a branching-time framework. Second, she assumes that the imperfective aspect quantifies over a partition of the restrictor-times (e.g. the *when*-clause times in (95), or a contextually-provided temporal restriction). Thus, (95) roughly means that the song-writing times are contained in a possible history which is regularly partitioned into intervals, each of which includes a pub-going time. Deo (2009: 493-494) ultimately explains the exception-tolerance of habituals using the flexibility introduced by the contextually-specified partition. For example, the partition in example (95) could group together certain song-writing events and separate others, leading to an imperfect match between song-writings and pub-goings.

Though this solution is extremely interesting in its own right, and is backed up by a sophisticated and precise analysis, it does not quite fit the novel data I present in Section 3.3. First, Deo requires that the partition that provides the modal quantifier domain must be *regular*. In other words, the intervals in the partition must be of equal measure. Assuming each song-writing takes around the same amount of time, Deo derives a result for (95) where either every song-writing corresponds to a pub-going (no exceptions), or every *n* song-writings correspond to a pub-going, for some context-dependent number *n* (regularly-grouped exceptions). The size of the partition determines which of these two kinds of readings is actually predicted. However, it seems that exceptions to habitual sentences can be quite irregular in general. For example, Bonnie's absences in Section 3.3 example (85) could be spaced out or clustered together. Moreover, the predicted reading, where every *n* song-writings correspond to a pub-going, does not seem like a natural reading of (95).

Second, and most importantly, on Deo's analysis there is no expectation that exception-tolerance should vary with the Question Under Discussion. It may be possible to relate the Question Under Discussion to the size of the partition via a pragmatic mechanism, but the required mechanism is not obvious, and pursuing such a fix is outside the scope of this study. On the other hand, the non-maximality account in Section 3.3 correctly predicts that exceptions are possibly irregular and dependent on the Question Under Discussion. Moreover, no special pragmatic mechanisms are required except those independently needed to account for definite plurals [Križ 2015].

3.4.3 Other related work

While Deo [2009] builds exception-tolerance into the theory of habituals by setting up a contextually-provided partition over times, and requiring that this partition match the times in the extension of the sentence radical, other approaches have attempted to weaken the truth-conditions of habituals by evaluating them with respect to primitive objects other than times and events.

Carlson [2008] analyzes both habitual and generic sentences using *patterns*. For Carlson, patterns are a primitive of the theory, and habituals and generics are true if and only if they are satisfied by a pattern. Patterns capture the non-accidental cooccurrence of events, and they naturally tolerate exceptions, unlike a restricted universal quantifier over times. Similarly, Bittner [2008] assumes the existence of *habits*, which are kinds at the event level. Habits, like patterns, are exception-tolerant by nature. These ideas are implemented very differently, and a detailed comparison would go beyond the scope of the present study. However, both approaches assume that there are some semantic primitives that are exception-tolerant by definition, and that these special objects serve as the truth-makers for habitual sentences, rather than more familiar objects such as times or events.

Though there may be independent reasons to include objects such as patterns or event-kinds in our models, neither account mentioned above is equipped to deal with the particular context-dependence of habitual sentences. As we have seen in Section 3.3, exceptions to habitual sentences are tolerated only if those exceptions are irrelevant to resolving the Question Under Discussion. This dimension of variation is unexpected on any analysis that attempts to weaken the truth conditions of habitual sentences by adding structured objects such as partitions, patterns, or habits.

Finally, a different approach, taken by Greenberg [2007], is to treat exception-tolerance in bare plural generics as a species of vagueness. The idea behind this approach is that generics are quantifiers, but their quantificational domain is vague. Thus, generics do not contain an exception-tolerant quantifier GEN, but instead contain a universal quantifier over a vague domain. Though Greenberg does not explicitly address habitual sentences like (95), one could imagine an extension of the vague quantifier domain theory to habituals.

In fact, Križ (2015: 40-42) notes that a unification of homogeneity and vagueness may be possible. However, there are two obstacles to such an approach. First, in borderline cases, vague predicates such as *tall* in (97) can be affirmed and denied of the same individual [Alxatib & Pelletier 2011; Ripley 2011]. In contrast, homogeneous predicates cannot be both affirmed and denied of the same plurality, even in cases like (98) when the predicate is true of a sizable proper subpart of that plurality.

(97) Bill is both tall and not tall. Križ 2015: 41:(137a)

(98) *Context: Half the books are in Dutch.* Križ 2015: 41:(137b)

The books are both in Dutch and not in Dutch.

Analogous examples with habitual sentences such as (99) are infelicitous, and therefore pattern with plural definites, rather than vague predicates.

(99) *Context: Ben shaves once a year.*

Ben both shaves and doesn't shave.

Second, Križ (2015: 42) notes that homogeneous predicates do not reproduce the Sorites paradox, though I omit the relevant examples for space reasons.

Most importantly, the vagueness approach to exception-tolerance does not straightforwardly explain the sensitivity of exceptions to the Question Under Discussion. Despite these arguments, a closer comparison of homogeneity and vagueness might be illuminating, especially since the origins of homogeneity and non-maximality are still not well-understood. Ultimately, a reductionist theory of homogeneity effects may be possible, but I leave such attempts to future work.

3.5 Conclusion

I have defended a view of habitual readings in English on which they are not produced by specialized aspectual operators, but instead arise naturally from independently motivated assumptions about plural predication. On this view, the exception-tolerance of habituals and the behavior of habituals under negation follow from the assumption that plural predication in general obeys homogeneity: the positive and negative extensions of temporal predicates must not overlap.

This theory has three advantages over existing alternatives, which I have outlined in Section 3.4. First, it is conceptually simple. It does not require expanding the ontology of natural language semantics beyond the standard assumptions of algebraic semantics [Krifka 1998]. As a result, the theory is modular, and can easily be extended to be compatible with event semantics and modal analyses of the imperfective (e.g. Deo 2009). Second, it naturally accounts for the facts in Section 3.3, which show that the exception-tolerance of habituals depends on the partition provided by the QUD. No other existing account captures this dependence. Third, it provides a unified perspective on disparate phenomena. The data in Section 3.2 shows that habitual sentences resemble plural definites, and that this resemblance is confirmed by multiple diagnostics. This resemblance follows from a deep symmetry between the semantics of these expressions.

4 | Modals and Non-maximality

In this chapter, I present a plural, referential analysis of weak necessity modals, developed in joint work with Paloma Jeretič. This proposal aims to explain parallels observed between weak necessity modals and definite pluralities of individuals on a number of properties that have previously been shown to characterize plural definites. The principal property we discuss is homogeneity: just like plural definites, weak necessity modals across languages are scopeless with respect to negation.

Unlike previous prominent analyses of weak necessity, ours has homogeneity as an intrinsic feature inseparable from the core semantics. In support of our analysis, we also show that weak necessity modals share other properties with plural definites, including homogeneity removal by quantifiers, QUD-sensitive exception tolerance, and truth value gaps diagnosed by responses to borderline sentences.

The second contribution of this paper is to propose a compositional account of the weak necessity modals that are morphologically derived from strong necessity modals (as observed in French, Javanese, and other languages), which builds on our new view of weak necessity modals as plural definites.

In Section 4.1, we present some background on weak necessity modals. In Section 4.2, we show the core data that provides support for our plural referential analysis of *should* and other weak necessity modals. In Section 4.3, we give our semantics for *should*, and in Section 4.4, we propose an extension to French and Javanese, in which weak necessity modals are compositionally built from strong necessity modals. In Section 4.5, we discuss a few previous analyses and compare them to our own, and in Section 4.6 we conclude.

4.1 Background on weak necessity modals

4.1.1 The characterization of weak necessity meaning

Weak necessity modals are necessity modals, like English *should*, that have been described to be weaker or more negotiable than strong necessity modals, like *must* or *have to*. This difference is illustrated below: *should* indicates a necessity that allows for exceptions, while *have to* does not allow for exceptions.

(100) a. If you want to go to Colegio de México, you should take a taxi. ...but you could also take a bus if you have all morning.

b. If you want to go to Colegio de México, you have to take a taxi. ...??but you could also take a bus if you have all morning.

This feeling of weakness (compared to strong necessity) is the primary focus of previous accounts, such as von Fintel & Iatridou [2008], Sloman (1970), and Horn (1972).

We focus on a different challenge posed by Weak Necessity modals, namely their homogeneous behavior under negation. Previous theories do not address the negation pattern, so these modals' weakness would appear to be independent from homogeneity.

On our account, homogeneity and weakness have the same source: The "weakness" is due to non-maximality, a general phenomenon encountered in plural predication, and one the unifying themes of this dissertation. As we will show, our account does no worse than any existing account when it comes to modeling the weakness of weak necessity modals, and improves on existing accounts by adding negation into the picture.

4.1.2 The expression of weak necessity cross-linguistically

Across languages, weak necessity can be formed in two different ways. The first way is as a morphologically non-decomposable lexical item, such as English *should* and *ought*.[1] In other cases, a weak necessity modal can be built from a strong necessity modal by adding additional morphology, whose nature varies across languages. In some languages, weak necessity modals are formed by combining a strong necessity modal with counterfactual morphology, as found for example in French, shown in (101) (see discussion of this strategy in von Fintel & Iatridou 2008).

(101) a. Strong necessity modal *devoir*

Tu **dois** partir.

you must go

You must go.

b. Weak necessity modal *devoir*+CF

Tu **devrais** partir.

you must.CF go

You should go.

Other languages express weak necessity by adding to a strong necessity modal a dedicated morpheme (i.e. not obviously related to any other semantic category in the language), as seen in Javanese, shown in (102), and related Malayo-Polynesian languages (see discussion of this strategy in Vander Klok & Hohaus 2020).

(102) a. Strong epistemic necessity *mesthi*

Yu Dur **mesthi** nek omah.

sister Dur EPIS.NEC at house

[1]In Old English *should* was an inflected form of *shall*. However, in the modern synchronic grammar of English, *should* is probably frozen.

'Dur must be at home.'

b. Weak epistemic necessity *mesthi*+NE

Yu Dur **mesthi-ne** nek omah.

sister Dur EPIS.NEC-NE at house

'Dur should be at home.'

4.2 Homogeneity in plural definites

A distinctive property of plural definites is their interaction with negation, in which they display *homogeneity effects*, meaning that they take obligatory apparent wide scope with respect to negation, even if they are syntactically lower [Fodor 1970; Löbner 1985].

(103) a. The guests are here, # but some of them aren't.

b. Every guest is here, # but some of them aren't.

In contrast with universal quantifiers, plural definites take obligatory apparent wide scope with respect to negation.

(104) a. The guests are not here, # but some of them are.

b. Every guest is not here, ✓ but some of them are.

Again, this pattern persists when the negation is located in a higher clause.

(105) a. I don't think the guests are here, # but some of them are.

b. I don't think every guest is here, ✓ but some of them are.

Plural definites do not actually take scope, but are interpreted directly as the arguments of predicates. The resulting reading is therefore not a "wide-scope" reading, but a *scopeless* one.

The surprising fact about weak necessity modals, which previous work has so far not analyzed, is that they too have the scopeless reading, testable by comparing same-clause negation to higher-clause negation.

4.2.1 Scopeless weak necessity modals

Under negation, weak necessity modals pattern like plural definites, where they must be interpreted as taking apparent wide scope. We observe a contrast between *should* and a strong necessity modals like *have to*, which can take scope below negation. Compare the pair in (106a-106b), where the modals are not under negation, to the pair (107a-107b), where negation produces a contrast.

(106) a. According to the rules, you should go,

but you are allowed not to go.

b. According to the rules, you have to go,

but you are allowed not to go.

We use an overt modal adverb *according to the rules* to fix the ordering source. This ensures that the most salient reading is the one on which both the modals *should*, *have to*, and *allowed to* are all evaluated with respect to the same ordering source.[2]

(107) a. According to the rules, you shouldn't go,

but you are allowed to go.

b. According to the rules, you don't have to go,

✓ but you are allowed to go.

[2]If we do not fix the ordering source, the example would look like *You shouldn't go, but you are allowed to go.* That example is felicitous under a reading where *should* is interpreted relative to some ordering source other than the one provided by the rules. (For example, we could be in a context where it is immoral or imprudent to follow the rules.)

Like with plural definites, the apparent wide scope persists when negation is extra-clausal. *Should* must have a wide scope interpretation, in contrast with *have to*, which takes scope below.

(108) a. According to the rules, I don't think you should go,

but you are allowed to go.

b. According to the rules, I don't think you have to go,

✓ but you are allowed to go.

(109) a. According to the rules, it's not the case that you should go,

but you are allowed to go.

b. According to the rules, it's not the case that you have to go,

✓ but you are allowed to go.

Note that some strong necessity modals like English *must* are also known to have wide scope readings with respect to negation, as in (110a) (Jeretič 2021, Homer 2015, Iatridou & Zeijlstra 2013). However, they differ from weak necessity modals in that they can be interpreted with low scope under extra-clausal negation, as shown in (110b).

(110) a. According to the rules, you mustn't go,

but you are allowed to go.

b. According to the rules, I don't think you must go,

✓ but you are allowed to go.

We take these examples as evidence that *must*'s wide scope in matrix sentences should not receive the same analysis as the wide scope of weak necessity modals (see Jeretič 2021 for an analysis of *must* that captures the difference between clausemate and non-clausemate negation).

As far as we know, all weak necessity modals have this apparent wide scope with respect to negation. We have checked this for: English *should*, French *falloir* and *devoir* with counterfactual (CF) marking, Javanese modals with NE marking, Russian *sledovat'/stoit'*, Swedish *bör*, Spanish *deber/tener que/hay que*+CF, Hungarian *kell*+CF, Portuguese *dever/haver de*+CF, Italian *occorrerre/bisognare/dovere*+CF, Dutch *moeten*+CF, Greek *prepi*+CF.

Since homogeneity effects are systematically found for weak necessity modals across languages, a suitable analysis of the meaning of weak necessity should capture this pattern. However, as we will show in Section 4.5, this is not the case for the most prominent previous analyses of weak necessity, where one has to account for homogeneity in an independent way, e.g. through positive polarity, as analyzed by Iatridou & Zeijlstra 2013 and Homer 2015. In contrast, the analysis proposed in this paper of weak necessity modals as plural definites has homogeneity as an intrinsic part of the meaning of a plurality, thus making the right prediction for the cross-linguistic picture.

To summarize, weak necessity modals can be compared to plural definites in their scopeless behavior with respect to negation, as shown in (109a) for *should* and (105a) for a plural definite. This scopeless behavior is to be contrasted with strong necessity modals and universal nominal quantifiers, which do not have such scopeless behavior, as shown in (109b) and (110b) for *have to* and *must*, and in (105b) for *every*.

These parallels suggest a unified analysis for weak necessity modals and plural definites, where the difference between 'should' and 'must' is the same as the one between 'the' and 'all'. On our analysis, weak necessity modals are referring expressions that denote pluralities of worlds, just like plural definite nominals denote pluralities of individuals. In the next sections, we provide additional support for this parallel by showing that weak necessity modals pass additional diagnostics for plural predication.

4.2.2 Homogeneity removal

We now look at a phenomenon called *homogeneity removal*, where a universal quantifier over the domain of individuals denoted by a plural definite "removes" its truth value gap. This is the case in (111b), where ‘all’ quantifies over the set of relevant guests (here ‘all’ is floating, but the effect is the same if it is not).

(111) a. The guests are not here, # but some of them are.

b. The guests are not **all** here, ✓ but some of them are.

Notice that, strictly speaking, nothing is being “removed” in (111). Quantifiers just interact with negation as expected, while plural definites are referring expressions, and therefore do not take scope with respect to negation.

There is a comparable phenomenon occurring with *should*, where introducing the modal quantifier adverb ‘necessarily’ removes homogeneity.

(112) a. The advice was that you shouldn’t go, # but that you can go.

b. The advice was that you shouldn’t **necessarily** go,
✓ but that you can go.

(113) a. According to the rules, you should not work from home...
but it is allowed.

b. According to the rules, you should not **necessarily** work from home... ✓ but it is allowed.

In both (112) and (113), we get a strong scopal contrast between the sentence with the quantifier ‘necessarily’ and the one without.

4.2.3 Exception tolerance

Plural predication tolerates exceptions in the right discourse contexts, in contrast with universal quantifiers. This kind of exception tolerance is also known as *non-maximality* [Dowty 1987; Schwarzschild 1996; Brisson 2003; Križ 2016; Malamud 2012]. Plural predication structures are accepted as true when the exceptions are not relevant to the QUD, or in other words, when an existential and a universal claim fall in the same cell in the partition made by the QUD.

In (114), we give an example of a context in which universal quantification is false, regardless of the QUD. In contrast, plural predication becomes true or false depending on whether the QUD asks about the truth of an existential claim (thus making irrelevant whether existential or universal quantification is true), like QUD1, or a universal claim, like QUD2.

(114) *C: 4 out of 6 students asked questions.*
QUD1: Was the class lively?/Did any students ask questions?
QUD2: Who all asked questions?/How many students asked questions?

a. The students asked questions. (QUD1: ✓; QUD2: #)

b. All the students asked questions. (QUD1: #; QUD2: #)

Weak necessity modals display the same pattern. While for plural definites, the "exceptions" are irrelevant individuals, for weak necessity modals the "exceptions" are irrelevant possibilities—worlds whose inclusion or exclusion in the plurality does not matter for the QUD.

(115) *C: One can get a perfect grade by doing most exercises correctly; doing all gives extra credit.*
QUD1: What is a way to get a perfect grade?
QUD2: What are the minimal requirements to get a perfect grade?

a. To get a perfect grade, you should do every exercise. (QUD1: ✓; QUD2: #)

b. To get a perfect grade, you have to do every exercise. (QUD1: #; QUD2: #)

In this context, a necessary condition to get a perfect grade is to do most exercises. Doing *all* of the exercises is not necessary. We observe that the strong necessity claim (115b) is infelicitous under any QUD. On the other hand, (115a) can be rescued by by a QUD like QUD1 above, which does not distinguish between cases in which the addressee does every exercise, and cases in which the addressee does most exercises, as long as they get a perfect grade.

4.2.4 Responses to indeterminate sentences

Another characteristic property of homogeneity is that outright denials of indeterminate sentences are infelicitous. Križ observes that in borderline cases (i.e. cases in which existential but not universal quantification is true), it is preferable to respond with *well*, rather than denial.

(116) *C: Mary talked to only some of the girls.*

a. A: #Mary talked to the girls.

B: #{No, That's not true}, only to some.

B: Well, only to some. A̲: #Mary talked to all of the girls.

B: {No, That's not true}, only to some.

B: #Well, only to some.

The same pattern applies to borderline cases with *should.*

(117) *C: Two doors lead to the living room; both are equally good options.*

a. A: #You should take the right door to go to the living room.

B: #{No, That's not true}, you don't have to, but you can.

B: Well, you don't have to, but you can.

b. A: #You must take the right door to go to the living room.

B: {No, That's not true}, you don't have to, but you can.

B: #Well, you don't have to, but you can.

Recall from the previous section that indeterminate sentences are felicitous in contexts where their exceptions are irrelevant to the QUD. Since this is equally true of sentences with negation, responses with *no* can be made felicitous by a friendly QUD. In those cases, the *well*-responses are also fine. So the cases that are crucial for distinguishing the present theory from bivalent theories are all cases with a strict QUD (i.e. where an existential and universal claim correspond to different answers).

4.3 Analysis

We have shown that the difference between weak and strong necessity modals appears to be empirically parallel to the difference between universal quantifier expressions and definite descriptions in the nominal domain. We base our analysis on this parallel, where weak necessity modal statements can be exactly paraphrased by plural nominals, with the same truth and falsity conditions, as shown in (118).

(118) a. *You should go.* $\simeq$ *You go in the best worlds.*

b. *You shouldn't go.* $\simeq$ *You don't go in the best worlds.*

(119) a. *You have to go.* $\simeq$ *You go in all best worlds.*

b. *You don't have to go.* $\simeq$ *You don't go in all best worlds.*

Homogeneity removal by quantifiers is also parallel:

(120) *You shouldn't necessarily go.* $\simeq$ *You don't go in all the best worlds.*[3]

We implement this analysis by directly translating the proposal by Križ [2016] of plural definite nominals as referential pluralities of individuals into the equivalent for worlds as paraphrased above.

[3] With the caveat that *all* can quantifier raise, in contrast with *necessarily*, and *have to* from the previous example. Here the target meaning is of course the narrow scope of the quantifier, revealing its lack of homogeneity.

4.3.1 Homogeneity for plural nominals

Križ [2016] follows standard accounts of plural reference where pluralities are mereological sums of individuals, and predicates have sums in their extension, as shown in (121).

(121) a. The windows are open. $\mathsf{open}(\bigoplus \mathsf{window})$

b. The windows are not open. $\neg\mathsf{open}(\bigoplus \mathsf{window})$

Plural predication is then assumed to have a homogeneity property, which amounts to requiring non-overlap between the positive and negative extensions of the predicate, defined as follows.

(122) **Homogeneity:** A plurality in the extension of a predicate $\lambda x.P(x)$ must not overlap with any plurality in $\lambda x.\neg P(x)$.

This means that in (121a), $\bigoplus\ulcorner\mathsf{window}\urcorner$ (the sum of all windows in the domain) must not overlap with anything that is not-open, which entails that *every* window is open. For (121b), $\bigoplus\ulcorner\mathsf{window}\urcorner$ must not overlap with anything open, which entails that *no* window is open. When *some but not all* of the windows are open in w, the plural predication structure is indeterminate in w—neither true nor false.

The semantics is trivalent. The indeterminate truth value is taken to be the source of *well*-responses, in cases where *yes* and *no* would be infelicitous. In discourse, this third truth value can be resolved to *true enough* in certain contexts, which is the source of the exception tolerance mentioned in Section (4.2.3). Finally, homogeneity removal occurs when a quantifier is introduced and operates on the set of individuals given by the plurality.

4.3.2 Weak necessity modals denote definite pluralities of worlds

Following standard Kratzerian modal semantics [Kratzer 1981], we assume a contextually supplied modal base and ordering source, which provides the modal domain D. D is the set of "best" worlds among the modal base, according to the ordering source. To reduce notational clutter, we do not explicitly separate the modal base and ordering source here, and just write D. Following Kratzer, a strong necessity modal is a universal quantifier: it asserts that the prejacent p (type `st`) is true in all the worlds in that domain.

We take weak necessity modals to be non-quantificational. Instead, a weak necessity modal refers to the sum of all worlds in the modal domain D. We then assume that the prejacent proposition has sums of worlds in its extension, which allows D to satiate the proposition's world argument, as shown in (124).

(123) $\text{should}_D := \bigoplus D$

(124) $$[\![\text{you should}_D \text{ go}]\!]^g = [\![\text{go(you)(should}_D)]\!]^g = [\![\text{go(you)}(\bigoplus D)]\!]^g$$
$$= \begin{cases} 1 & [\![\text{go(you)}(w)]\!]^g = 1 \text{ for all } w \in g(D) \\ 0 & [\![\text{go(you)}(w)]\!]^g = 0 \text{ for all } w \in g(D) \\ \star & \text{otherwise} \end{cases}$$

This analysis is based on a trivalent semantics for plural predicates, as described in the previous chapters. It therefore immediately gives us the properties discussed in Section 4.2 common to individual and world pluralities: homogeneity and homogeneity removal, preference for *well*-responses to indeterminate weak necessity modals, as in (117), and exception tolerance of weak necessity modals (see Appendix for formal details).

4.4 Deriving weak necessity from strong necessity

As shown in Section 4.1, in many languages, weak necessity is derived from a strong necessity modal and an additional morpheme, with some differences in the meaning of this morpheme across languages. We propose a compositional analysis that derives weak necessity from strong necessity, and is equipped to capture the differences between languages.

4.4.1 Picking out a witness set of a quantifier

With an analysis of weak necessity as a plurality of worlds, we can derive it from combining a strong necessity modal with an additional operator X, defined in (125). We appeal to the notion of 'minimal witness set' (inspired by, but not identical to, the analysis of weak necessity in Vander Klok & Hohaus 2020), defined as a minimal set that makes a quantifier true, in (125b).

(125) $X := \lambda M_{<st,st>}. \bigoplus \iota W[W \in WIT(M)]$

where:

a. For a set of sets $\mathcal{S}$, $\iota S[S \in \mathcal{S}]$ picks out the unique set in $\mathcal{S}$ if $\mathcal{S}$ is a singleton, and is undefined otherwise.

b. W is a *minimal witness set* of M iff $W \in M$ and $\neg\exists W' \subset W : W' \in M$.

c. $WIT(M)$ is defined as the set of minimal witnesses for the modal M.

In words, X is an operator that picks out the unique smallest set that makes a quantifier true and takes the mereological sum of its elements. Applied to a universal quantifier, it simply picks out the domain of that quantifier, which is exactly what we need to derive a weak necessity modal, which is a domain of worlds, from a strong necessity modal, which is a universal quantifier over that domain.

Therefore, we apply X to a strong necessity modal, defined in (126a), to derive a plurality of worlds as shown in (126b).

(126) a. $\text{must}_D := \lambda p.\forall w \in D.p(w)$

b. $X(\text{must}_D) = \bigoplus \iota W[W \in WIT(must)] = \bigoplus D$

Thus, X is an operator that allows us to derive a sum of worlds from a universal quantifier. We now show how this operator, or versions of it, predicts the distribution of the weak necessity-forming morpheme in specific languages.

4.4.2 Cross-linguistic variation in the morpheme deriving weak necessity

Vander Klok & Hohaus 2020 show that the Javanese morpheme NE, which forms weak necessity modals, is only found on necessity modals, but not possibility. This restriction can be seen in the following example .

(127) Vander Klok & Hohaus 2020: pg. 2: (3)

a. Aku iso ngelangi.
1SG CIRC.POS AV.swim
'I can swim.'

b. *Aku iso-ne ngelangi.
1SG CIRC.POS-NE AV.swim

Our proposal captures this restriction: X picks out the unique witness set of a quantifier, and returns the plurality associated with that set. Indeed, an existential quantifier has many minimal witness sets, as many as there are elements in its domain. However X is only defined if there is one minimal witness set. Therefore X can apply to universal quantifiers. This captures the observed distribution for Javanese NE.

Additional support for NE in Javanese picking out a unique minimal witness set is that the morpheme NE is also used as a definiteness marker for nominals. This syncretism is found in many related Malayo-Polynesian languages, including Madurese and Indonesian (see [Vander Klok & Hohaus 2020] and references therein). We might therefore expect that NE expresses definiteness applying to an individual or sum of individuals, and that the minimal witness set operator is a null morpheme that arises in the case of modals in order to avoid a type clash. We leave investigation of this idea to future work.

In other languages (e.g. French, and many others, see [von Fintel & Iatridou 2008]), the morpheme deriving weak necessity is the morpheme used for counterfactual statements, and can also apply to possibility modals, as in the following French examples.

(128) a. Tu devrais partir.
you must.CF leave
You should leave.

b. Tu pourrais partir.
you can.CF leave
You could leave.

The current definition of X does not capture the felicity of counterfactually-marked possibility modals. However, while we defined X to be parallel to definite plural nominals, i.e. defined for a *unique* minimal witness set, definiteness is in fact not a necessary component of homogeneity in Križ's analysis, and all that is in fact needed for homogeneity is plural reference.

Therefore, in order to capture the typology, we propose that this counterfactual marking in French and analogous languages picks out a witness set, but without the uniqueness requirement (and arguably without the minimality requirement; we leave further refinements to future work).

(129) $\textsc{cf} := \lambda M_{<st,st>}. \bigoplus W$ for some $W \in WIT(M)$

This allows for the morpheme to apply to both possibility and necessity modals, and still derive homogeneity effects with weak necessity modals. This proposal is in line with [Schlenker 2004], who takes counterfactual marking to be a distal demonstrative pointing to a set of worlds.

4.5 Previous analyses don't capture homogeneity effects

We now go over prominent previous analyses of weak necessity modals and show that none are able to account for the basic homogeneity diagnostic sentence, repeated below in (130), where a negated weak necessity modal is incompatible with an existential claim.

(130) According to the rules, you shouldn't go, # but you can.

We also discuss other properties of weak necessity and how they are better captured with the pluralities of worlds approach rather than alternative analyses.

4.5.1 Domain restriction

A first prominent analysis of weak necessity is the domain restriction approach, most notably represented by von Fintel & Iatridou [2008]. This type of analysis is based in a standard Kratzerian modal framework, where strong necessity modals like *must* are universal quantifiers over the set of best worlds according to an ordering source. Weak necessity modals are also universal quantifiers, but quantify over a subset of what the domain of strong necessity modal would be, by picking out the best worlds according to a second ordering source. In particular, if *must* quantifies over the best accessible worlds according to ordering source g_1, *should* quantifies over the best according to g_2 of the best according to g_1. Without any additional assumptions, this analysis does not capture the basic homogeneity effects: *should* is a universal quantifier, so its negation is wrongly predicted to be compatible with an existential quantifier.

4.5.1.1 Homogeneity removal data

The domain restriction approach to weak necessity modals has a hard time accounting for the homogeneity removal data, repeated in .

(131) According to the rules, you should not **necessarily** work from home...

✓ but it is allowed.

If the secondary ordering source is what triggers homogeneity, one could stipulate that *necessarily* functions as a modifier that removes the secondary ordering source. But *necessarily* has its independent life as a modal quantifier, and it functions as a homogeneity remover in other constructions, like conditionals and habituals. It is unclear why *necessarily* should function both as a secondary ordering source remover and a homogeneity remover in cases that are far from being analyzed as having something comparable to a secondary ordering source (e.g. conditionals, habituals).

4.5.2 Proportional quantifier approaches

A second approach is to treat *should* as analogous to *most* (Horn 1989). Under this approach, *should* is to *must* what *most* is to *all*. In other worlds, *should*(p) says that most of the best accessible worlds are p-worlds.

(132) a. ⟦You must go⟧ = *In all of the best accessible worlds, you go.*

b. ⟦You should go⟧ = *In most of the best accessible worlds, you go.*

In this analysis, the negation of *should* should again be compatible with an existential quantifier (as shown by a felicitous continuation of the paraphrase in [b] *but in some, you don't*). This analysis therefore does not capture *should*'s homogeneity.

Furthermore, one advantage of this approach, at least at first glance, is that it appears to be able to capture the exception tolerance of *should*. However, it does not allow the degree of exception-tolerance to depend on the QUD. Instead, it depends on the exact proportion of p-worlds to $\neg p$-worlds in an abstract modal space, and it is unclear how a proportional quantifier could change its meaning relative to the QUD.

4.5.3 Degree-based approaches

Finally, we find a class of analyses that can be qualified as degree-based, found in Lassiter [2011],Lassiter [2017], and Portner & Rubinstein [2016]. Under these approaches, weak necessity modals introduce a probability (for epistemics) or utility (for priority modals) function μ over propositions, whose result is then compared to some contextually-supplied standard d.

(133) $[\![$should p$]\!]$ iff $\mu(p) > d$

Such an analysis does not capture homogeneity effects: the negation of *should* as defined in (133) is compatible with an existential quantifier, as long as $d \neq 0$. The defender of the degree-based approach could push back on this. For example, the value of the contextual standard could shift under negation. We think that a plausible version of this response could be developed, but its development is outside the scope of the present study, so we leave further discussion on this point to future work.

4.6 Conclusion

Weak necessity modals show behavior parallel to plural nominals, and should receive a parallel analysis. On the basis of homogeneity diagnostics shown above, we conclude that weak necessity modals denote pluralities of worlds, in contrast to strong necessity modals, which are genuine quantifiers. Our analysis uniquely both captures homogeneity and homogeneity removal. The homogeneity removal data, in particular, are challenging for accounts of weak necessity modals like von Fintel & Iatridou [2008], which rely on a secondary ordering source to restrict the domain of the modal. We also provide an account of exception tolerance, which can be favorably compared to previous theories that address exceptions. Finally, we provide the most explicit compositional account of the derivation of weak necessity from strong necessity, which can capture some systematic cross-linguistic patterns, as well as certain kinds of variation in the marking of weak necessity that would be unexpected on other accounts.

5 Conclusion

5.1 Summary of the main points of the dissertation

In this dissertation, I have presented three case studies in which the theory of homogeneity and non-maximality is applied to novel problem areas, with interesting results. In Chapter 2, I formally develop Križ's (2015) theory of homogeneity and non-maximality in conditionals, and present novel arguments in its favor. The idea that *if*-clauses satisfy homogeneity requirements has been around since at least von Fintel [1997]. The novel contribution here was to show that not only is it *possible* to model Sobel sequences using non-maximal plural reference to worlds (as shown by Križ 2015), but that the non-maximality approach has a distinct advantage over the basic variably strict analysis. On a variably strict analysis, a sequence of the form (i) ⌜if A, C⌝; (ii) ⌜if A and B, not C⌝ can be jointly true because the two conditionals can quantify over disjoint sets of worlds. However, the problem with the variably strict analysis is that it predicts that the sequence above entails that the antecedent ⌜A and B⌝ is false, an incorrect prediction pointed out by Willer [2017]. The advantage of the non-maximality approach is that it avoids this problem. The non-maximality approach does not require felicitous Sobel sequences to be true, only true enough (in the sense of Križ [2015]).

This approach raises further questions because it is possible to combine Križ's Sufficient Truth theory with either a strict semantics for conditionals or a variably strict semantics. I show that the non-maximality approach to Sobel sequences, when combined with a strict semantics for bare indicative conditionals, makes correct predictions for so-called reverse Sobel sequences. The variably strict theory augmented with non-maximality has too many knobs to turn, leading to some subtle problems in the treatment of reverse Sobel sequences. This novel theoretical finding has relevance both for researchers interested in non-maximality applied to conditionals, and for researchers interested in the tradeoffs between strict and variably strict theories of indicative conditionals.

In Chapter 3, I show that bare habituals and non-quantificational temporal adverbs display homogeneity effects. Though this was suggested in previous work by Ferreira [2005], the novel contribution of this work was to show that the same constructions also display non-maximality. Moreover, I provide novel theoretical arguments showing that the analysis in terms of non-maximality is preferable to previous analyses of imprecision in habituals. In particular, Deo [2009], the most sophisticated previous analysis, relies on a context sensitive modal. The context sensitive modal analysis does not capture the effect of the QUD on judgments about habitual sentences, which are consistent with non-maximality. The modal analysis also makes the assumption that the habitual events occur regularly with a fixed frequency, an assumption that turns out to be too strong in some cases. I show that the predictions of non-maximality provide a better empirical fit.

Finally, in Chapter 4, I present a theory of weak necessity modals like English *should*, according to which they denote pluralities formed by taking the sum of a set of accessible worlds. The theory is novel, and was developed in co-authored work with Paloma Jeretič. While previous work on these modals has focused on their weakness (relative to strong modals like *must* and *have to*), we broaden the empirical picture by considering their behavior under negation. We show that not only does *should* appear to take wide scope with respect to negation (like *must*), but surprisingly, it appears to take wide scope over a negation in a higher clause. In this respect, *should* is different from *must*, but similar to plural definites. We conclude that the wide scope of *should* is only apparent, and that *should* is actually *scopeless*.

Our plural, referential analysis easily captures the scopelessness of *should* and other weak necessity modals in other languages. It also captures the weakness of these modals at least as well as existing approaches, which do not consider the scopelessness at all. Finally, we show that *should* becomes strong again when combined with the modal slack-regulator *necessarily*, which is independently known to remove homogeneity in conditionals, generics, and temporal adverb constructions. This last fact is difficult to explain for other theories, and would likely require some extra stipulations. In the present theory it follows immediately, as a natural consequence of Križ's (2015) framework. Thus, our work on modals presents novel empirical arguments that any theory of weak necessity modals must consider, along with a novel theory that explains the data.

These case studies provide concrete and plausible alternatives to the standard view that these constructions involve universal quantifiers across the board. Methodologically, homogeneity and non-maximality can be used as diagnostics for probing the logical forms of unfamiliar expressions. In the case of temporal adverbs, we observe a sharp boundary between quantificational and referential adverbs. Interestingly, many of the quantificational adverbs contain words or morphemes that do quantificational work in other domains.

5.2 Directions for future work

5.2.1 Bare plural generics

Generics are known to tolerate exceptions. However, the degree of exception-tolerance can be quite extreme, especially in so-called relative generics like *Mosquitos carry West Nile Virus*. Furthermore, generics can also communicate inviolable laws, as in *Bishops move diagonally*. Križ [2015] suggests that bare plural generics involve homogeneity and non-maximality, but leaves it open (i) whether homogeneity comes from a plurality of events, or from a plurality of individuals, or both, and (ii) whether non-maximality is the only factor in explaining the exception-tolerance of generics, or only one among several.

A Sufficient Truth theory of generics should be able derive both relative and inviolable readings of generics from a uniform trivalent semantics. The degrees of exception tolerance would arise from the Question Under Discussion, which is partially dependent on the genre of discourse under study. Relative readings arise in contexts where the QUD is indifferent to the exceptions, no matter how numerous they are. For example, when warning someone about the dangers of mosquitos, non-dangerous mosquitos are not relevant. Inviolable readings arise in contexts where the QUD requires maximally exact answers. So, in the context of a mathematical proof, the QUD should be such that exceptions are not tolerated. It remains to be seen whether this context-dependence can provide a basis for a theory of generics based on non-maximality, and I see this as a promising direction for future work.

5.2.2 Focus-driven QUD accommodation

In new work in progress, I consider novel data involving plural definites under focus. I observe that non-maximal readings are especially easy when the plural definite associates with a focus-sensitive operator, such as *only* or *even*. No existing theory of homogeneity directly predicts any interaction between non-maximality and the presence of *only*/*even*. However, such interactions are not entirely surprising, since focus is independently thought to be QUD-sensitive (Roberts 2012, Beaver & Clark 2008, a.o.).

(134) *Context: Five professors and five students were watching a talk. The students were quiet, but three of the professors asked questions.*
Only the professors asked questions.

(135) *Context: Five professors, five graduate students, and five undergraduate students are watching a talk together. The discussion is very active: all the professors and graduate students ask*

questions. In addition, two out of the five undergraduates ask questions.

That talk was very lively. Even the undergraduates asked questions.

The implications of this finding go beyond plural definite noun phrases. So-called *only if*-conditionals have the surprising property that ⌜p only if q⌝ does not entail ⌜p if q⌝. Schlenker [2004] and Križ [2015]—and this dissertation—claim that bare conditionals are plural predication structures, in the sense that *if*-clauses are plural definite descriptions of worlds, and the consequent (main clause) is a predicate of worlds that accepts plural arguments. The pattern of judgments I describe for plural definites under *only* should easily extend to cases where *only* associates with an *if*-clause.[1] To bolster this argument, I show that conditionals under *even* have a similar pattern to *only*-if conditionals, extending the parallel between conditionals and definite descriptions even further. This observation is also new, as far as I know.

The key idea is that the focus structure of of a sentence can implicitly change or constrain the inferred QUD via *focus-driven QUD accommodation.* Focus-driven QUD accommodation is a process in which uncertainty about the QUD is resolved by accommodating a QUD that is congruent to the focus structure of the last-uttered sentence. I aim to show that Sufficient Truth can be evaluated with respect to an accommodated QUD. For example, if a speaker utters sentence *S* in a context with no explicit QUD, then *S* can be true enough with respect to its accommodated QUD, which will be the simplest question that is focus-congruent to *S*.

QUD accommodation was first discussed in these terms by Roberts (2012: pp. 39-40, 47-48) who describes two instances of QUD accommodation driven by focus structure. In each of Roberts's two cases, the accommodated QUD is superquestion of some QUD that is already on the stack. Since Roberts's initial discussion of this idea, the notion has been invoked in various contexts. For example, Bledin & Rawlins [2019] assume that QUD accommodation can occur to satisfy the requirements of *what if*-questions. Cooper & Larsson [2010] study cases of QUD ac-

[1] von Fintel's (1997) original analysis of *only if* proposed that conditionals have a homogeneity presupposition, but did not adopt a plural referential theory of conditionals. The present analysis builds on von Fintel's original observations by adding non-maximality into the picture.

commodation in a dialogue corpus. Onea (2016: Ch 4: pp. 168-175) studies QUD accommodation in the context of potential questions, and explicitly talks about the role of focus marking.

The most thorough discussion, as far as I know, is found in Beaver & Clark [2008], who lay out specific principles constraining QUD accommodation. However, QUD accommodation is a complex and general issue that goes beyond the present study on focus and non-maximality, and more empirical work is needed to understand its constraints.

Some more recent works on *only if*-conditionals [Herburger 2019; Bassi & Bar-Lev 2018] have chosen a different strategy. They argue that conditionals have an underlying existential semantics, according to which ⌜if p, q⌝ means that *some* of the closest *p*-worlds are *q*-worlds. The existential quantifier is strengthened via a free-choice-like exhaustification mechanism, which does not apply under *only* or under negative quantifiers. These theories focus on conditionals under *only* and conditionals in the scope of negative quantifiers, and they do not account for the parallel behavior of *only* and *even*.

In the new work in progress, I aim to lay out an explicit theory of focus-driven QUD accommodation, show how it applies to non-maximal plural definites under focus, and compare the focus-based theory to existential theories such as Bassi & Bar-Lev [2018]. Through this investigation, I hope to further develop the framework of non-maximality and push it beyond the core cases.

A | Appendix to Chapter 3

Appendix A: Trivalent Type Theory

In this section I provide the formal details of the trivalent type theory used by [Križ 2015], which I adapt for this work. I will not repeat Križ's presentation here. Instead I will review the most important ideas, and direct the reader to Križ (2015: Chapter 2) and [Lepage 1992] for further details. The key idea is that partial functions—functions that are indeterminate for some values—can be ordered according to their indeterminacy, and this ordering follows from an ordering on the domain of truth values $D_{\mathtt{t}}$.

Let $D_{\mathtt{t}} = \{0, 1, \star\}$ and let $\leqslant_{\mathtt{t}}$ be a partial order on $D_{\mathtt{t}}$. Intuitively, $x \leqslant_{\mathtt{t}} y$ should be read as x *is at most as indeterminate as* y. The relation $\leqslant_{\mathtt{t}}$ is given by (i) $x \leqslant_{\mathtt{t}} x$ and (ii) $0, 1 \leqslant_{\mathtt{t}} \star$. This gives a join semilattice in which (crucially) $\star$ overlaps with both 0 and 1, but 0 and 1 do not overlap. We will now extend this ordering to all functions.

The set of **types** $\mathscr{T}$ is the smallest set $\mathscr{T}$ such that $\mathtt{e}, \mathtt{i}, \mathtt{t} \in \mathscr{T}$, and if $\sigma, \tau \in \mathscr{T}$, then $\langle\sigma\tau\rangle \in \mathscr{T}$. I abbreviate types right-associatively, so that $\langle\sigma\tau\rangle \equiv \sigma\tau$, $\langle\rho\langle\sigma\tau\rangle\rangle \equiv \rho\sigma\tau$, and $\langle\langle\rho\sigma\rangle\tau\rangle \equiv (\rho\sigma)\tau$. Let the ordering $\leqslant_{\mathtt{t}}$ be as before, and let $\leqslant_{\mathtt{e}}$ and $\leqslant_{\mathtt{i}}$ stand for the usual mereological parthood relation on individuals $D_{\mathtt{e}}$ and time intervals $D_{\mathtt{i}}$ respectively.

By induction on types $\tau \in \mathscr{T}$, we can recursively extend the orderings on these basic domains to an ordering on an arbitrary domain D_τ in the following way. Let ρ be any basic type, i.e. $\rho \in \{\mathtt{e}, \mathtt{i}, \mathtt{t}\}$, so that $\leqslant_\rho$ is already defined. Thus, in the base case, where $\tau = \rho$, we have already defined the relation $\leqslant_\rho$. Now, suppose $\tau = \sigma\rho$, where $\sigma \in \mathscr{T}$ is an arbitrary type. In this case, for functions $f, g \in D_{\sigma\rho}$, we say that $f \leqslant_{\sigma\rho} g$ if and only if $f(x) \leqslant_\rho g(x)$ for all $x \in D_\sigma$. For the most general case, suppose $\sigma_0, \sigma_1, ..., \sigma_n \in \mathscr{T}$ are arbitrary types, and $\tau = \sigma_0\sigma_1...\sigma_n\rho$, where $\rho \in \{\mathtt{e}, \mathtt{i}, \mathtt{t}\}$ as before. For functions $f, g \in D_{\sigma_0\sigma_1...\sigma_n\rho}$, we say that $f \leqslant_{\sigma_0\sigma_1...\sigma_n\rho} g$ if and only if $f(x) \leqslant_{\sigma_1...\sigma_n\rho} g(x)$ for all $x \in D_{\sigma_0}$.

The implementation is complex, but the idea is simple. For example, consider two functions $f, g \in D_{\mathtt{et}}$ (imagine that f and g are the denotations of two intransitive verbs). Suppose $f(x)$ is determinate (1 or 0) on all individuals $x \in D_{\mathtt{e}}$, but $g(a) = \star$ for some particular individual $a \in D_{\mathtt{e}}$, and suppose further that $g(x) = f(x)$ for all $x \neq a$. In this case, $f \leqslant_{\mathtt{et}} g$, because $f(x) \leqslant_{\mathtt{t}} g(x)$ for all $x \in D_{\mathtt{e}}$. However, for example, (i) if f and g are indeterminate for *distinct* inputs, or (ii) if they assign 0 and 1 respectively to the same individual, then they will not be ordered by $\leqslant_{\mathtt{et}}$.

Given orderings $\leqslant_\tau$ for each type τ, we can also define a general notion of overlap. This notion of overlap is the key ingredient in Križ's formalization of homogeneity, given below in (136).

(136) Definition of Homogeneity — Križ 2015: 53:Def. 2.9

For any type $\sigma \in \mathscr{T}$, let $\circ_\sigma$ denote **overlap** with respect to the ordering $\leqslant_\sigma$ on the domain D_σ; that is to say, $x \circ_\sigma y$ if and only if there is a $z \in D_\sigma$ such that $z \leqslant_\sigma x$ and $z \leqslant_\sigma y$.

A function $f : D_\sigma \to D_\tau$ is **homogeneous** iff for all $x, y \in D_\sigma$, $(x \circ_\sigma y) \to f(x) \circ_\tau f(y)$.

For a simple example, suppose $f \in D_{\mathtt{et}}$ is a *homogeneous* function, and $a, b, c \in D_{\mathtt{e}}$ are individuals. Recall that $0 \circ_{\mathtt{t}} \star$ and $1 \circ_{\mathtt{t}} \star$, but $\neg(0 \circ_{\mathtt{t}} 1)$. According to the homogeneity generalization, if $f(a \oplus b) = 1$, then any $x \in D_{\mathtt{e}}$ that overlaps with $a \oplus b$ will be mapped to either 1 or $\star$ by f. Formally, if $x \circ_{\mathtt{e}} (a \oplus b)$, then homogeneity forces either $f(x) = 1$ or $f(x) = \star$; $f(x)$ may not be 0. In particular, $f(a)$ must be 1 or $\star$ and $f(a \oplus b \oplus c)$ must be 1 or $\star$. But, since c does not overlap with $a \oplus b$, $f(c) = 0$ is possible (as long as $f(a \oplus b \oplus c) \neq 1$).

Before moving on, let me comment on some differences between the presentation here and the presentation in Križ (2015: Ch. 2). First, Križ only uses types e and t, and he only defines the ordering $\leqslant_\tau$ for types ending in $\mathtt{t}$. I instead extend $\leqslant_\tau$ to the whole type hierarchy, treating the mereological parthood relations $\leqslant_{\mathtt{e}}$ and $\leqslant_{\mathtt{i}}$ on a par with the indeterminacy ordering $\leqslant_{\mathtt{t}}$.[1] More importantly, Križ discusses extensions of the trivalent type theory to cover collective predicates, non-homogeneous predicates, and non-monotonic quantifiers. I do not adopt these extensions here, so as not to obscure my key points, but it would certainly be possible to implement them.

Appendix B: The Language $\mathscr{L}$

I use a typed λ-calculus $\mathscr{L}$ with three types, truth values, entities, and times. The **syntax** of the λ-language $\mathscr{L}$ is entirely standard, so I will review it only briefly. In what follows, ρ, σ, τ are metavariables over types, and α, β, a, b, p, q, s, t, x, y are all metavariables over terms. For each type $\tau \in \mathscr{T}$, let $\mathscr{L}_\tau$ stand for the set of terms of type τ in $\mathscr{L}$. Let Var_τ stand for the set of variables of type τ, and let Con_τ stand for the set of constants of type τ. Then, $\mathscr{L}$ is the smallest set satisfying rules (i-x): (i) [Variables and Constants] $Var_\tau \cup Con_\tau \subseteq \mathscr{L}_\tau$, (ii) [Application] If $\alpha \in \mathscr{L}_{\sigma\tau}$ and $\beta \in \mathscr{L}_\sigma$, then $\alpha(\beta) \in \mathscr{L}_\tau$, (iii) [Abstraction] if $x \in Var_\sigma$ and $\alpha \in \mathscr{L}_\tau$, then $(\lambda x.\alpha) \in \mathscr{L}_{\sigma\tau}$.

We have the usual **logical symbols**: (iv) [Negation] If $p \in \mathscr{L}_{\mathtt{t}}$, then $(\neg p) \in \mathscr{L}_{\mathtt{t}}$, (v) [Disjunction, Conjunction, Implication] If $p, q \in \mathscr{L}_{\mathtt{t}}$, then $(p \wedge q), (p \vee q), (p \rightarrow q) \in \mathscr{L}_{\mathtt{t}}$, (vi) [Quantifiers] If $x \in Var_\tau$ and $p \in \mathscr{L}_{\mathtt{t}}$, then $(\forall x[p]), (\exists x[p]) \in \mathscr{L}_{\mathtt{t}}$.

We also have the following **non-logical symbols**: (vii) [Equality] if $\alpha, \beta \in \mathscr{L}_\tau$, then $(\alpha = \beta) \in \mathscr{L}_{\mathtt{t}}$, (viii) [Parthood, Overlap] if $\sigma \in \{\mathtt{e}, \mathtt{i}\}$ and $\alpha, \beta \in \mathscr{L}_\sigma$, then $(\alpha \preceq \beta), (\alpha \circ \beta) \in \mathscr{L}_{\mathtt{t}}$, (ix) [Sum] if $\sigma \in \{\mathtt{e}, \mathtt{i}\}$ and $\alpha \in \mathscr{L}_{\sigma\mathtt{t}}$, then $\bigoplus \alpha \in \mathscr{L}_\sigma$, and (x) [Precedence] if $s, t \in \mathscr{L}_{\mathtt{i}}$, then $(s \ll t) \in \mathscr{L}_{\mathtt{t}}$.

Now we define the **semantics** of $\mathscr{L}$. Let $\mathscr{M}$ be the set of all models. M is a **model** if $M = \langle \mathscr{I}_M, D_{\mathtt{e}}, \leqslant_{\mathtt{e}}, D_{\mathtt{i}}, \leqslant_{\mathtt{i}}, \ll_{\mathtt{i}}, W \rangle$, where (i) $\mathscr{I}_M$ is an interpretation function, (ii) $\langle D_{\mathtt{e}}, \leqslant_{\mathtt{e}} \rangle$, is an atomic

[1] One could, using the definitions in this Appendix, define homogeneous functions to individuals or times, but the applications are not clear. This is just a convenience for me, since both Križ and I only force homogeneity for types ending in $\mathtt{t}$ anyway (see (141) in Appendix B).

join semilattice of individuals, (iii) $\langle D_{\mathrm{i}}, \leqslant_{\mathrm{i}} \rangle$ is a non-atomic join semilattice of times, (iv) $\langle D_{\mathrm{i}}, \ll_{\mathrm{i}} \rangle$ is a partially-ordered set of times (where $\ll_{\mathrm{i}}$ is the precedence ordering), and (v) W is a set of possible worlds. Independently of the choice of model M, the domain of **truth values** is always defined as $D_{\mathrm{t}} = \{0, 1, \star\}$, and is always ordered by $\leqslant_{\mathrm{t}}$ (defined in Appendix A). For any types $\sigma, \tau \in \mathscr{T}$, the functional domain $D_{\sigma\tau}$ is a set of functions from D_σ to D_τ.

The **denotation** function $[\![\cdot]\!]^w_{M,g}$ maps terms in $\mathscr{L}_\tau$ to model-theoretic objects (functions) in D_τ, for any type $\tau \in \mathscr{T}$, according to the rules in (137,138,139).

(137) Basic Semantic Rules

a. (Variables) If $x \in Var$, then $[\![x]\!]^w_{M,g} = g(x)$.

b. (Constants) If $\mathbf{a} \in Con$, $[\![x]\!]^w_{M,g} = \mathscr{I}_M(\mathbf{a})$.

c. (Application) If $\alpha \in \mathscr{L}_{\sigma\tau}$ and $\beta \in \mathscr{L}_\sigma$, then $[\![\alpha(\beta)]\!]^w_{M,g} = [\![\alpha]\!]^w_{M,g}([\![\beta]\!]^w_{M,g})$.

d. (Abstraction) If $\lambda x.\alpha \in \mathscr{L}_{\sigma\tau}$, then $[\![\lambda x.\alpha]\!]^w_{M,g}$ is a function from $D_\sigma \to D_\tau$, given by $u \mapsto [\![\alpha]\!]^w_{M,g[x/u]}$.

(138) Rules for Logical Symbols

a. (Negation) If $p \in \mathscr{L}_{\mathrm{t}}$, then $[\![\neg p]\!]^w_{M,g} = \begin{cases} 1 & [\![p]\!]^w_{M,g} = 0 \\ 0 & [\![p]\!]^w_{M,g} = 1 \\ \star & \text{otherwise} \end{cases}$

b. (Conjunction) If $p, q \in \mathscr{L}_{\mathrm{t}}$, then $[\![p \wedge q]\!]^w_{M,g} = \begin{cases} 1 & [\![p]\!]^w_{M,g} = [\![q]\!]_{M,g} = 1 \\ 0 & [\![p]\!]^w_{M,g} = 0 \text{ or } [\![q]\!]^w_{M,g} = 0 \\ \star & \text{otherwise} \end{cases}$

c. (Disjunction) If $p, q \in \mathscr{L}_{\mathtt{t}}$, then $[\![p \vee q]\!]^w_{M,g} = \begin{cases} 1 & [\![p]\!]^w_{M,g} = 1 \text{ or } [\![q]\!]^w_{M,g} = 1 \\ 0 & [\![p]\!]^w_{M,g} = [\![q]\!]_{M,g} = 0 \\ \star & \text{otherwise} \end{cases}$

d. (Implication) If $p, q \in \mathscr{L}_{\mathtt{t}}$, then $[\![p \rightarrow q]\!]^w_{M,g} = [\![(\neg p) \vee q]\!]^w_{M,g}$

(139) Rules for Non-Logical Symbols

a. (Equality) $[\![\alpha = \beta]\!]^w_{M,g} = 1$ iff $[\![\alpha]\!]^w_{M,g} = [\![\beta]\!]^w_{M,g}$, and 0 otherwise.

b. (Precedence) For $s, t \in \mathscr{L}_{\mathtt{i}}$, $[\![\alpha \ll \beta]\!]^w_{M,g} = 1$ iff $[\![s]\!]^w_{M,g} \ll_{\mathtt{i}} [\![t]\!]^w_{M,g}$, and is 0 otherwise.

c. (Parthood) For $\sigma \in \{\mathtt{e}, \mathtt{i}\}$, and $\alpha, \beta \in L_\sigma$, $[\![\alpha \leqslant \beta]\!]^w_{M,g} = 1$ iff $[\![\alpha]\!]^w_{M,g} \leqslant_\sigma [\![\beta]\!]^w_{M,g}$, and equals 0 otherwise.[2]

d. (Overlap) For $\sigma \in \{\mathtt{e}, \mathtt{i}\}$, and $\alpha, \beta \in L_\sigma$, $[\![\alpha \circ \beta]\!]^w_{M,g} = 1$ iff $[\![\alpha]\!]^w_{M,g} \circ_\sigma [\![\beta]\!]^w_{M,g}$, i.e. there exists some $z \in D_\sigma$ such that $z \leqslant_\sigma [\![\alpha]\!]^w_{M,g}$ and $z \leqslant_\sigma [\![\beta]\!]^w_{M,g}$, and equals 0 otherwise.

e. (Sum) For $\sigma \in \{\mathtt{e}, \mathtt{i}\}$, and $\alpha \in L_{\sigma\mathtt{t}}$, $[\![\bigoplus \alpha]\!]^w_{M,g}$ is the unique sum of the set $\left\{x \in D_\sigma \mid [\![\alpha]\!]^w_{M,g}(x) = 1\right\}$.

We impose the following constraints on admissible models:

(140) $M = \langle \mathscr{I}_M, D_{\mathtt{e}}, \leqslant_{\mathtt{e}}, D_{\mathtt{i}}, \leqslant_{\mathtt{i}}, \ll_{\mathtt{i}}, W \rangle$ is an admissible model iff:

a. (CEM) Both individuals $\langle D_{\mathtt{e}}, \leqslant_{\mathtt{e}} \rangle$ and times $\langle D_{\mathtt{i}}, \leqslant_{\mathtt{i}} \rangle$ satisfy all the axioms of Classical Extensional Mereology (Champollion 2017: 13-17).

b. (Precedence and Overlap) All and only non-overlapping pairs of time intervals are in the precedence relation, i.e. $\forall x, y \in D_{\mathtt{i}} [(x \ll_{\mathtt{i}} y) \vee (y \ll_{\mathtt{i}} x) \leftrightarrow \neg(x \circ y)]$.

[2]Though I define $\leqslant_\tau$ for all types τ in Appendix A, I assume that the symbol $\leqslant$ in $\mathscr{L}$ only denotes mereological parthood.

With all this in place, we can formally state the homogeneity constraint, where *homogeneous* is defined in (136) in Appendix A. This constraint requires that functions of all types ending in $\mathtt{t}$ are homogeneous.[3]

(141) Homogeneity Constraint

For all $\sigma_1, .., \sigma_n \in \mathscr{T}$, the domain $D_{\sigma_1 \ldots \sigma_n \mathtt{t}}$ must be a set of homogeneous functions.

Appendix C: A Formal Fragment

This appendix contains an explicit fragment with derivations for a few interesting sentences. $\langle \cdot \rangle$ is a function from object language expressions (parsed English phrases) to terms in $\mathscr{L}$. The compositional order is given by the syntactic parse in the following way: If ϵ and δ are object-language expressions, then $\langle\ [\ \epsilon\ [\ \delta\ ]\ ]\ \rangle = \langle\ [\ [\ \delta\ ]\ \epsilon\ ]\ \rangle = \langle \epsilon \rangle(\langle \delta \rangle)$.

(142) a. $\langle \textsc{past} \rangle = \lambda T_{\mathtt{it}} \lambda s_{\mathtt{i}}.T(s) \wedge \partial(r \leqslant s) \wedge \partial(r \ll u)$

b. $\langle \textsc{pres} \rangle = \lambda T_{\mathtt{it}} \lambda s_{\mathtt{i}}.T(s) \wedge \partial(r \leqslant s) \wedge \partial(r \leqslant u)$

(143) $\langle$Annie runs$\rangle$ = $\langle$[[[Annie] run] -s]$\rangle$ = $\langle \textsc{pres} \rangle(\langle \text{run} \rangle(\langle \text{Annie} \rangle)) = \lambda t_{\mathtt{it}}.\partial^{g,r}_{\textsc{pres}}(t) \wedge \mathsf{run}(\mathsf{a})(t)$

(144) $\langle \text{on} \rangle = \lambda A_{(\mathtt{it})\mathtt{t}} \lambda T_{\mathtt{it}}.\mathsf{match}(M) \wedge A(\lambda u_{\mathtt{i}}.T(M(u)))$

(145) $\langle \text{every Saturday} \rangle = \lambda R_{\mathtt{it}}.\forall s_{\mathtt{i}}[\mathsf{saturday}(s) \rightarrow R(s)]$

(146) $\langle$[on [every Saturday]]$\rangle$ = $\langle \text{on} \rangle(\langle \text{every Saturday} \rangle) = \lambda T_{\mathtt{it}}.\mathsf{match}(M) \wedge \forall s_{\mathtt{i}}[\mathsf{saturday}(s) \rightarrow T(M(s))]$

[3] Križ [2015] discusses how to account for non-homogeneous functions as well, but introducing them further complicates the logic, so I set them aside.

(147) $\langle$[[Annie runs] [on [every Saturday]]]$\rangle = \mathsf{match}(M) \land \forall s_{\mathtt{i}}[\mathsf{saturday}(s) \rightarrow \partial^{g,r}_{\text{PRES}}(M(s)) \land \mathsf{run(a)}(M(s))]$

(148) $\langle$Saturdays$\rangle = \lambda R_{\mathtt{it}}.R[\bigoplus(\mathsf{saturday})]$

(149) $\langle$[on [Saturdays]]$\rangle = \lambda T_{\mathtt{it}}.\mathsf{match}(M) \land T(M[\bigoplus(\mathsf{saturday})])$

(150) $\langle$[[Annie runs] [on [Saturdays]]]$\rangle$ =

$\mathsf{match}(M) \land \partial^{g,r}_{\text{PRES}}[M(\bigoplus(\mathsf{saturday}))] \land \mathsf{run(a)}[M(\bigoplus(\mathsf{saturday}))]$

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